Optimal Monetary Policy with an Uncertain Cost Channel

Peter Tillmann
University of Bonn

this version: April 2007

Abstract: The cost channel of monetary transmission describes a supply side effect of interest rates on firms’ cost. Previous research has found this effect to vary over time and across countries. Moreover, the cyclical nature of financial frictions is likely to amplify the cost channel. This paper derives optimal monetary policy in the presence of uncertainty about the cost channel. In a minmax approach, the central bank derives an optimal policy plan to be implemented by a Taylor rule. It is shown that uncertainty about the cost channel leads to an attenuated interest rate setting behavior. The model is able to reconcile the response coefficients of optimal interest rate rules with those obtained from empirical studies of U.S. monetary policy. These results are obtained for a plausible degree of uncertainty about the cost channel. In contrast, uncertainty about other key parameters cannot explain the attenuated interest rate setting behavior that is found in the literature.

Keywords: parameter uncertainty, minmax, cost channel, optimal monetary policy, Taylor rule

JEL classification: E52, E31

1University of Bonn, Institute for International Economics, Lennéstr. 37, D-53113 Bonn, E-mail: tillmann@iwi.uni-bonn.de
I thank Andreas Schabert and Michael Evers for very helpful comments on an early draft of the paper and seminar participants at the Universities of Bonn and Dortmund for stimulating discussions. The DFG network "Quantitative Macroeconomics" provided financial support.
1 Introduction

Monetary policymakers typically face uncertainty about key relationships driving the economy. Uncertainty potentially disguises the true economic model that governs the economy, disturbs parameters, and introduces noise into the data series available to the policymaker. This paper focuses on the consequences of uncertainty about the supply side effect of monetary policy, which is known as the cost channel of monetary transmission. Firms are assumed to pay the factors of production before they receive revenues from selling their products. Hence, they rely on borrowing working capital from financial intermediaries. The effect of interest rate changes on the cost of acquiring and holding working capital is referred to as the cost channel.

To the extent that deregulation, financial integration, and, most importantly, the cyclical nature of financial frictions affect the availability of working capital, the central bank is likely to be not perfectly informed about the true size of the cost channel. Complete knowledge would require a full understanding of how commercial banks pass-through interest rate changes to their customers and how credit conditions in general change after, say, a monetary tightening.

Recently, Ravenna and Walsh (2006) derive optimal monetary policy in the presence of a cost channel. This paper introduces uncertainty into their model and solves for optimal monetary policy that is robust to model misspecifications drawing on the recent work on optimal monetary policy under uncertainty, e.g., Giannoni (2002, 2005), Onatski and Stock (2002), and Onatski and Williams (2003). A policymaker determines optimal policy before the uncertainty about the cost channel is resolved. The central bank follows a minmax approach and seeks to minimize the worst possible loss that could occur due to parameter misspecification.

The aim of this paper is twofold. First, the paper analyzes the normative consequences for optimal policy under cost channel uncertainty. We seek to contribute to the ongoing debate as to whether uncertainty makes interest rate setting less or more aggressive. A recent strand of the literature derives consequences of uncertainty for interest rate setting. Giannoni (2002) models uncertainty about the slopes of the Phillips curve and the Euler equation for output and derives a robust minmax policy that is implemented by a simple instrument rule. His results imply that the policymaker responds more strongly to inflation than under certainty. Likewise, Stock (1999), Onatski and Stock (2002), and Onatski and Williams (2003) deduce optimal Taylor rules under
uncertainty. The literature typically finds that uncertainty is likely to lead to more vigorous interest rate setting behavior. Policy no longer obeys the “Brainard principle” (Brainard, 1967), stating that multiplicative parameter uncertainty should motivate a cautious policy stance, i.e. an attenuated interest rate adjustment.²

Second, the paper pursues a related question and asks whether uncertainty about the cost channel contributes to the understanding of the actual interest rate setting behavior of the Federal Reserve. A common finding points to the discrepancy between theoretically derived optimal interest rate rules and empirically supported Taylor rules.³

The latter typically implies a much smaller interest rate response to contemporaneous inflation than the former. Some authors propose that taking uncertainty into account might reconcile the aggressive optimal policy stance with the attenuated observed policy. However, in an important contribution, Rudebusch (2001) shows that, roughly speaking, uncertainty has only a minor impact on interest rate setting and cannot explain this discrepancy.

By introducing uncertainty about the cost channel we address both the normative and the positive question. A crucial result of the paper is that uncertainty about the cost channel can explain the attenuated policy stance. An uncertain policymaker should overestimate the quantitative importance of the cost channel when setting interest rates. Hence, the larger the degree of uncertainty about the cost channel, the smaller the interest rate response to inflation. In this sense, the policymaker is less aggressive than under certainty.

Monetary policy under multiplicative uncertainty, as in Brainard’s (1967) classical result, is generally less aggressive than under certainty while interest rate setting under minmax policy is generally more aggressive as in Giannoni’s (2002) paper. The presence of a cost channel bridges these opposing views. Cost channel uncertainty attenuates interest rate adjustment even under minmax policy. We also show that taking the uncertain cost channel into consideration yields a robust optimal Taylor rule which matches the empirically estimated rule for post-1982 Federal Reserve policy.

The remainder of this paper is organized as follows. Section two integrates uncertainty about the cost channel of monetary transmission into an otherwise standard

²The results of Tetlow and von zur Mühlen (2001) show that the effect of uncertainty on interest rate setting might be less clear-cut than the distinction between attenuation and anti-attenuation.

³Taylor (1993) describes U.S. monetary policy employing a simple interest-rate feedback rule, in which the interest rate is set in response to inflation and output gap movements.
New Keynesian model and derives implications for optimal minmax monetary policy under parameter uncertainty. In section three, the instrument rule that implements the optimal minmax equilibrium, i.e. the robust optimal Taylor rule, is derived and contrasted with empirically observed policy rules. Section four finally concludes.

2 Robust monetary policy with an uncertain cost channel

Recent empirical research by Barth and Ramey (2001) and others draws attention to the transmission of monetary impulses to the economy through the cost channel, which describes a supply-side effect of monetary policy that augments the conventional demand-side channel. To the extend that firms must pay the factors of production before they receive revenues from selling their products, they rely on borrowing working capital from financial intermediaries. Monetary policy therefore impacts on the cost side of the economy. Higher interest rates translate into higher cost of working capital and induce a rise in inflation. Recently, Ravenna and Walsh (2006) and Chowdhury, Hoffmann, and Schabert (2006) integrate a cost channel into an otherwise standard New-Keynesian model of the business cycle and show that the presence of a cost channel is tantamount to a direct effect of interest rates on the inflation rate within a forward-looking Phillips curve.

2.1 The model

We adopt a standard forward-looking monetary model of the business cycle. The conventional representation of the New Keynesian Phillips curve is extended to allow interest rates to impact on the supply-side of the economy by raising firms’ marginal cost and, thus, the inflation rate. We draw on recent work by Christiano, Eichenbaum, and Evans (2005), Ravenna and Walsh (2006), and others, who introduce cost of working capital into a general equilibrium model.

Assume a slightly simplified version of Ravenna and Walsh’s (2006) model. The forward-looking Phillips curve (1) and the IS curve (2) represent log-linearised equi-

\[\text{See, among others, Woodford (2003) for a deeper analysis and the complete derivation of this family of models based on optimizing households and firms under monopolistic competition and nominal rigidities.}\]
librium conditions of a simple sticky-price general equilibrium model

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \left[ (\sigma + \eta) x_t + \psi_i t \right] \]  \hspace{1cm} (1)

\[ x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + u_t \] \hspace{1cm} (2)

where \( \pi_t \) is the inflation rate, \( x_t \) the output gap, \( i_t \) the risk-free nominal interest rate controlled by the central bank, and \( E_t \) is the expectations operator. The output gap is defined as the difference between actual output and the flexible-price output level conditional on the interest rate being constant. All variables are expressed in percentage deviations from their respective steady state values. The discount factor is denoted by \( \beta < 1 \), \( \sigma \) is the coefficient of relative risk aversion, and \( \kappa \), the slope coefficient of the Phillips curve, depends negatively on the degree of price stickiness. The demand shock \( u_t \) is described by \( u_t \sim \mathcal{N}(0,1) \). The coefficient \( \psi \) describes the direct impact of interest rates on inflation and, thus, the strength of the cost channel. If \( \psi = 0 \), the model collapses to the standard New Keynesian model considered by Clarida, Galí, and Gertler (1999) or Woodford (2003). In the presence of a cost channel of monetary transmission, however, changes in interest rates directly propagate into inflation dynamics.\(^5\)

Monetary policy is assumed to set interest rates in order to minimize the welfare loss due to sticky-prices which is described in terms of inflation volatility and output gap volatility weighted by the parameter \( \lambda \geq 0 \)

\[ \min_{i_t} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t, \quad \text{with} \quad \mathcal{L}_t = \pi_t^2 + \lambda x_t^2 \] \hspace{1cm} (3)

When formulating optimal policy, the policymaker does not know the true values of the parameter vector \( \vartheta = (\kappa, \psi) \) with \( \vartheta \epsilon \Theta \). The only information the central bank has available is that the coefficients lie in a range of parameters. In particular, the interval from which nature draws these parameters is given by

\( \kappa \epsilon [\kappa_l, \kappa_h] \quad \text{and} \quad \psi \epsilon [\psi_l, \psi_h] \)

where \( \kappa_h > \kappa_l > 0 \) and \( \psi_h > \psi_l > 0 \). The representative household, on the contrary, is perfectly informed about \( \vartheta \). Uncertainty about \( \kappa \) reflects imperfect information about

---

\(^5\)In the presence of a cost channel, the demand shock introduces a policy trade-off between output and inflation variation. The policymaker cannot set the interest such that the output gap is insulated from demand shocks without causing fluctuations in the rate of inflation.
the true degree of nominal rigidity. Uncertainty about $\psi$ reflects the uncertain nature of financial conditions for firms that want to borrow working capital. The sources of cost channel uncertainty warrant some explanations, which are provided in the next subsection. In a later subsection, we will also examine the implications of uncertainty about $\sigma$.

### 2.2 The uncertain cost channel

Empirical estimates of the cost channel vary substantially across countries as well as over time. Ravenna and Walsh (2006) obtain an estimate for $\psi$ of 1.276 for the U.S., which is, however, surrounded by considerable uncertainty. An interval of two standard errors around their estimate spans a range of plausible cost channel coefficients of $\psi \in [0.277, 2.261]$. With an alternative set of instruments, these authors obtain a considerably larger estimate for $\psi$ of 1.915. Moreover, Chowdhury, Hoffmann, and Schabert (2006) obtain a $\psi$ coefficient ranging from about 0.2 for France to 1.3 for the U.S. and the UK and 1.5 for Italy. Barth and Ramey (2001) split the sample and find a reduced importance of the cost channel in the post-1979 period in their VAR study. Tillmann (2006) estimates the augmented Phillips curve given in (1) in a rolling-window regression and obtains a series of cost channel coefficients over the last 25 years. He shows that $\psi$ follows a U-shaped pattern. The cost channel was most important in the pre-Volcker period and less important in the Volcker-Greenspan era. Recently, however, the cost channel regained quantitative importance.

What lies behind the shifts in the cost channel coefficient? Barth and Ramey (2001) point to financial innovations and deregulation as two forces that crucially affect the availability of working capital. Another important reason could be added: Given that financial intermediaries play a crucial role in propagating interest rate shocks to the cost-side of the economy and, finally, to inflation, financial frictions affect inflation dynamics in the presence of a cost channel and amplify monetary policy impulses.

In the following, we do not take a particular stand on the determinants of this pattern. Nevertheless, it appears that changing financial markets accompanied by the role of financial frictions might explain the apparent instability in the cost channel coefficient.

As a bottom line, however, it is clear that the policymaker faces uncertainty about the

---

6See Rabanal (2006) for a paper that finds only limited evidence for the existence of a cost channel.
true nature of the cost channel.\textsuperscript{7} Whatever the source, uncertainty about the cost channel might be an important factor behind actual monetary policy implementation. In addition, a vast literature documents the imperfect and asymmetric nature of the interest rate pass-through from policy-controlled rates to lending rates.\textsuperscript{8} If lending rates adjust sluggishly to interest rate increases, the central bank is likely to be uncertain about the true tightness of credit market conditions reflected in an uncertain cost channel.

2.3 A minmax approach to parameter uncertainty

In this model, the policy maker plays a zero sum game against a fictitious evil agent who sets the parameter vector such that the welfare loss is maximized. At the beginning of the period, the central bank commits to a policy \( f \in F \) before the parameter vector \( \vartheta \in \Theta \) is known. The class of rules \( F \) contains the feasible set of non-inertial policy rules, i.e. rules that do not imply interest rate inertia or other forms of history-dependent monetary policy. The central bank wants to set policy such as to minimize the worst possible loss that can occur due to parameter perturbations. This optimal robust policy is denoted by \( f^* (\vartheta^*) \) and is defined by

\[
\min_{f \in F} \left\{ \max_{\vartheta \in \Theta} E \left[ L_t (f (\vartheta), \vartheta) \right] \right\} \tag{4}
\]

At the same time, the central bank commits to implementing this non-inertial policy plan \( f \) by setting the interest rate, its policy instrument, according to a simple contemporaneous interest rate rule of the form proposed by Taylor (1993).\textsuperscript{9} This rule is characterized by a parameter vector \( \Phi = (\phi_\pi, \phi_x) \), \( \phi_\pi \geq 0 \) and \( \phi_x \geq 0 \), with

\[
i_t = \phi_\pi \pi_t + \frac{1}{4} \phi_x x_t \tag{5}
\]

and relates the interest rate to inflation and output gap movements. After that, the demand shock realizes and parameter uncertainty is resolved. Given the policy plan \( f \) and

\textsuperscript{7}Levin et al. (2005) consider parameter uncertainty in a model including the cost channel, but do not consider uncertainty about the cost channel.

\textsuperscript{8}On the sluggish dynamics of the pass through from policy-controlled interest rates to retail rates see Mojon (2000), Hofmann and Mizen (2004), de Bondt (2005), and the references therein. Gambacorta (2004) sheds light on the cross-sectional differences in the interest rate pass-through.

\textsuperscript{9}See Kara (2002), Giannoni (2002), Tetlow and von zur Mühlen (2001) and Onatski and Williams (2003) for similar approaches to model optimal minmax policy within a Taylor rule framework.
the true parameter values, equilibrium inflation and equilibrium output are realized and the interest rate is set according to the specified instrument rule. We denote the resulting minmax equilibrium by $\pi_t (f^*(\vartheta^*), \vartheta^*)$ and $x_t (f^*(\vartheta^*), \vartheta^*)$, where $f^*(\vartheta^*) \in F$ is the robust monetary policy plan and $\vartheta^* \in \Theta$ maximizes the welfare loss. Thus, following Giannoni (2002), Kara (2002) and others, the steps towards the solution are the following: First, derive the policy plan $f(\vartheta)$ that is optimal for a given parameter vector $\vartheta$. Second, find the parameter vector $\vartheta^*$ that does maximum damage to the representative agent’s utility. Finally, choose an optimal instrument rule that implements the resulting equilibrium $\pi_t (f^*(\vartheta^*), \vartheta^*)$, $x_t (f^*(\vartheta^*), \vartheta^*)$ and $i_t (f^*(\vartheta^*), \vartheta^*)$.

2.4 Deriving optimal policy

The central bank’s task is to derive the optimal policy plan $f(\vartheta)$ and the corresponding optimal instrument rule to implement this plan. To solve this problem, we will compute the optimal policy given the central banks reference parameter set. Given this policy, we then determine the parameter set that maximizes the welfare loss.

We know that an optimal non-inertial policy plan links output and inflation as follows

$$x_t = -f(\vartheta)\pi_t$$

(6)

where $f(\vartheta) > 0$ is a function of the model parameters. Given the policy plan, the output variance is therefore given by

$$\text{var}(x_t) = f^2(\vartheta)\text{var}(\pi_t)$$

(7)

The IS curve implies $i_t = -\sigma(x_t - u_t)$, which can be inserted into the inflation adjustment equation (1) to obtain

$$\text{var}(\pi_t) = \left[\frac{\kappa\sigma\psi}{(1 + \kappa f(\vartheta)(\sigma + \eta) - \kappa\sigma f(\vartheta)\psi)}\right]^2 \text{var}(u_t)$$

(8)

We analyze discretionary policy which takes expectations about future variables as given. Hence, the objective function can be compactly written as a function of the policy plan $f(\vartheta)$

$$L_t = \frac{\kappa^2\sigma^2\psi^2}{(1 + \kappa f(\vartheta)(\sigma + \eta) - \kappa\sigma f(\vartheta)\psi)^2 (1 + \lambda f^2(\vartheta))}$$

(9)
A robust control approach to policy under model uncertainty as advocated in Hansen and Sargent (2005) focuses on the maximum loss for any given policy $f$.

In a first step, we determine $f$. Given the parameter vector $\vartheta$, the central banker formulates optimal policy by minimizing the welfare loss

$$
\min_f \left\{ \frac{\kappa^2 \sigma^2 \psi^2}{\left(1 + \kappa f (\sigma + \eta) - \kappa \sigma f \psi\right)^2} (1 + \lambda f^2) \right\} \quad (10)
$$

The first order condition is given by

$$
f^* (\vartheta) = f^* (\kappa, \psi) = \frac{\kappa}{\lambda} \sigma (1 - \psi) + \eta \quad (11)
$$

where $f^* (\kappa, \psi)$ is the optimal non-inertial policy plan that minimizes the welfare loss.\(^{10}\)

The second step pertains to the determination of the worst-case parameter constellation. The central bank faces uncertainty about the true realization of the cost channel coefficient $\psi$ and the slope coefficient $\kappa$. Given a particular policy $f^*$, the central bank must find the worst-case realization which optimal robust policy wants to account for.

The decision maker solves the problem

$$
\max_{\kappa, \psi} \left\{ \frac{\kappa^2 \sigma^2 \psi^2}{\left(1 + \kappa f^* (\sigma + \eta) - \kappa \sigma f^* \psi\right)^2} (1 + \lambda f^*^2) \right\} \quad (12)
$$

The first order condition are given by

$$
\frac{\partial L_t}{\partial \kappa} = 1 > 0 \quad \text{and} \quad \frac{\partial L_t}{\partial \psi} = \psi (1 + \kappa f^*) > 0 \quad (13)
$$

Let $\kappa^* = \arg \max_{\kappa} \left\{ \frac{\kappa^2 \sigma^2 \psi^2}{\left(1 + \kappa f^* (\sigma + \eta) - \kappa \sigma f^* \psi\right)^2} (1 + \lambda f^*^2) \right\}$ denote the worst-case belief of the central bank. Since, $\partial L_t/\partial \kappa > 0$, $\kappa^* = \kappa_h$. This is the parameter perturbation that the central bank considers the worst possible misspecification against which policy should shield the economy. In order to stabilize demand shocks, the central bank must contract the economy. If $\kappa$ is high, the required output adjustment has large inflationary consequences.

Let $\psi^* = \arg \max_{\psi} \left\{ \frac{\kappa^2 \sigma^2 \psi^2}{\left(1 + \kappa f^* (\sigma + \eta) - \kappa \sigma f^* \psi\right)^2} (1 + \lambda f^*^2) \right\}$ denote the worst-case belief of the central bank about a misspecified cost channel coefficient. Note that $\partial L_t/\partial \psi$ is positive for any value of $f^*$ considered in this paper. Hence, $\psi^* = \psi_h$. An ambiguity-averse central banker should deliberately overestimate the true size of the cost channel.

\(^{10}\)The policy plan is identical to the one derived in Ravenna and Walsh (2006).
and set policy accordingly in order to minimize the welfare consequences of cost channel misspecification. As in Giannoni (2002) and other papers, the worst-case parameter set corresponds to the boundaries of the constrained parameter set $[\kappa_l, \kappa_h]$ and $[\psi_l, \psi_h]$. To summarize, the candidate worst case beliefs of the central bank are given by $\vartheta^* = (\kappa^*, \psi^*) = (\kappa_h, \psi_h)$ and the robust policy plan is given by

$$f^*(\vartheta^*) = f^*(\kappa_h, \psi_h) = \frac{\kappa_h}{\lambda}[\sigma(1 - \psi_h) + \eta]$$

(14)

Given this plan, the equilibrium responses of output, inflation, and the interest rate are given

$$\pi_t(f^*(\vartheta^*), \vartheta^*) = \Omega_{\pi} u_t, \quad x_t(f^*(\vartheta^*), \vartheta^*) = \Omega_{x} u_t, \quad i_t(f^*(\vartheta^*), \vartheta^*) = \Omega_{i} u_t$$

with

$$\Omega_{\pi} = \frac{\kappa_h \sigma \psi_h}{1 + \kappa_h f^*(\kappa_h, \psi_h)(\sigma + \eta) - \kappa_h f^*(\kappa_h, \psi_h)\sigma \psi_h}$$

(15)

$$\Omega_{x} = \frac{-f^*({\kappa_h, \psi_h}) \kappa_h \sigma \psi_h}{1 + \kappa f^*(\kappa_h, \psi_h)(\sigma + \eta) - \kappa_h f^*(\kappa_h, \psi_h)\sigma \psi_h}$$

$$\Omega_{i} = \frac{\sigma + \sigma \kappa_h f^*(\kappa_h, \psi_h)(\sigma + \eta) - \kappa_h f^*(\kappa_h, \psi_h)\sigma \psi_h}{1 + \kappa_h f^*(\kappa_h, \psi_h)(\sigma + \eta) - \kappa_h f^*(\kappa_h, \psi_h)\sigma \psi_h}$$

These minmax equilibrium realizations are implemented by setting interest rates according to an instrument rule characterized by the response coefficients $\Phi = (\phi_{\pi}, \phi_{x})$, which we derive in subsequent sections.

### 2.5 The effect of uncertainty

How do equilibrium inflation, output gap, and interest rate dynamics respond to changes in parameters and, in particular, to changes in the degree of uncertainty? To gauge how the reactions to shocks given in (15) change when the potential parameter misspecification increases, we calibrate the model to the U.S. economy as follows.

Ravenna and Walsh (2006) estimate an augmented Phillips curve with U.S. data and obtain an estimate for $\psi$ of 1.276, which serves as a reference value for the cost channel coefficient in this paper. These estimates are surrounded by considerable uncertainty. An interval of two standard errors around their estimate spans a range of plausible cost channel coefficients of $\psi \in [0.277, 2.261]$. With an alternative set of instruments, these authors obtain an estimate for $\psi$ of 1.915. Our measures of $\psi_h$ are set to 1.80 or 1.90 in order to prevent $f$ given in (11) from becoming negative. It appears quite plausible that the policymaker does not go so far as to consider values of $\psi_h$ which let him combat inflationary shocks by actually lowering interest rates. Given the empirical
findings in Ravenna and Walsh (2006), our worst-case $\psi$ implies a very modest degree of uncertainty.

Ravenna and Walsh’s (2006) estimates imply a value of $\kappa = 0.10$, while Christiano, Eichenbaum, and Evans (2005) find $\kappa = 0.2$. We follow Surico (2006) and set $\kappa = 0.15$. A grave misspecification of the Phillips curve is represented by $\kappa_h = 0.30$, which corresponds to the estimates obtained by Roberts (1995). The inverse of the real interest rate sensitivity of aggregate demand is set to $\sigma = 1$. Finally, setting $\eta = 1$ and $\lambda = 0.25$ follows standard practice in the literature. All parameters are summarized in table (1).

<table>
<thead>
<tr>
<th>Table 1: Parameter values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
</tr>
<tr>
<td>0.15</td>
</tr>
</tbody>
</table>

How does the strength of interest rate adjustment depend on the degree of parameter uncertainty? The resulting $\Omega_i$, $\Omega_x$, and $\Omega_y$ coefficients are plotted in figures (4) to (6) as a function of $\kappa_h$ and $\psi_h$. The interest rate responds less to the demand shock if uncertainty about the cost channel increases, see figure (4). In this respect, the popular “Brainard (1967) principle” holds. The central bank fears that interest rate steps have large inflationary consequences and, hence, refrains from aggressive stabilization. It follows that inflation, as depicted in figure (5), is allowed to fluctuate more as uncertainty about the cost channel increases. Output fluctuations, see figure (6), are subdued if cost channel uncertainty is large. This is because inflation is stabilized less under high cost channel uncertainty and, at the same time, $f^*(\bar{y}^*)$ is close to zero for large values of $\psi_h$.

In contrast, uncertainty about $\kappa$ raises the interest rate response to shocks. Monetary policy becomes more aggressive as the uncertainty about the slope of the Phillips curve increases. Under the simplifying assumption that $\psi_h = \psi = 1$, for example, $\partial \Omega_i / \partial \kappa = \sigma > 0$. In this case, the anti-attenuation result found by Giannoni (2002) and others holds. Put differently, cost channel uncertainty tends to make interest rate responses more cautious while uncertainty about the degree of price rigidity leads the policymaker to adjust interest rate more strongly.

Monetary policy under multiplicative uncertainty, as in Brainard’s (1967) classical result, is generally less aggressive than under certainty while interest rate setting under
worst-case policy is generally more aggressive as in Giannoni’s (2002) paper. Here, cost channel uncertainty attenuates interest rate adjustment even under minmax policy. Clearly, the welfare loss increases sharply as the degree of uncertainty about both coefficients increases, see figure (7). Furthermore, the fact that welfare deteriorates monotonically if \( \kappa^\ast \) or \( \psi^\ast \) increases, confirms that the minmax equilibrium given in (15) is indeed a global Nash equilibrium, i.e. there is no \( \hat{\vartheta} \) satisfying \( L_t(f^\ast(\hat{\vartheta}), \hat{\vartheta}) < L_t(f^\ast(\vartheta^\ast), \vartheta^\ast) \).

3 Optimal Taylor rules under uncertainty

The equilibrium realizations of output, inflation and the interest rate are only a function of the current value of the demand disturbance. Following Giannoni (2002), this optimal non-inertial policy plan can be implemented by an instrument feedback rule. Importantly, the degree of uncertainty about model parameters affects the coefficients of the instrument rule.

3.1 Implementing optimal policy with a Taylor rule

The policymaker implements the optimal non-inertial plan by setting the interest rate according to a simple contemporaneous interest rate rule of the form proposed by Taylor (1993). This rule is characterized by a parameter vector \( \Phi = (\phi_\pi, \phi_x) \), \( \phi_\pi \geq 0 \) and \( \phi_x \geq 0 \), with

\[ i_t = \phi_\pi \pi_t + \frac{1}{4} \phi_x x_t \]  

(16)

Although we refer to \( \phi_\pi \) and \( \phi_x \) as response coefficients, we do not assume that the policymaker literally responds to inflation and output gap movements as required by (16). Rather, the central bank sets interest rates according to that rule in order to realize the optimal policy plan and the resulting equilibrium characterized in (15). Hence, the Taylor rule serves as a convenient way to describe policy in terms of observables rather than in terms of shocks. The task is to derive the optimal \( \Phi \) that implements this plan. Note that the output coefficient is divided by four to make coefficients comparable with empirical observations that typically focus on annual rates rather than the model’s quarterly rates.

Inserting the equilibrium responses of inflation, output, and the interest rate to the
demand shock known from (15) into the Taylor rule gives the feasibility constraint

$$\Omega_i = \phi_\pi \Omega_\pi + \phi_x \frac{1}{4} \Omega_x$$  \hfill (17)$$

The optimal Taylor rule response coefficient to inflation as a function of $\phi_x$ and the distorted parameters is given by

$$\phi_\pi = \frac{\Omega_i}{\Omega_\pi} - \phi_x \frac{1}{4} \frac{\Omega_x}{\Omega_\pi}$$  \hfill (18)$$

or, using (15), can be written as

$$\phi_\pi = \frac{1}{\Omega_\pi} + \left( \frac{(\kappa^*)^2}{\kappa^* \psi^*} \right) \frac{\sigma (1 - \psi^*) + \eta}{\sigma + \eta} \left( \frac{\sigma}{\lambda} \left( 1 - \psi^* \right) + \eta \right) \frac{1}{4} \phi_x$$  \hfill (19)$$

for $\psi > 0$. Any vector $\Phi = (\phi_\pi, \phi_x)$ satisfying (19) implements the non-inertial minmax equilibrium described by (15).

How does the interest rate response to inflation, $\phi_\pi$, react if the degree of uncertainty changes? First, it is immediately apparent that

$$\frac{\partial \phi_\pi}{\partial \psi^*} < 0$$  \hfill (20)$$

Hence, $\phi_\pi(\psi_h) < \phi_\pi(\psi)$. A central bank being uncertain about the strength of the cost channel will respond less aggressively to inflation. This is a key result of this paper.\[11\] The interest rate response to inflation within an optimal robust Taylor rule decreases when uncertainty about the cost channel of monetary transmission becomes larger. Hence, interest rate setting is attenuated under uncertainty.

Therefore, uncertainty about the cost channel is one key candidate evaluated in the next section that can possibly reconcile theoretically derived coefficients (under certainty) with empirically observed behavior. Second, uncertainty about $\kappa$ affects the Taylor rule coefficient according to

$$\frac{\partial \phi_x}{\partial \kappa^*} = -\frac{1}{\psi^* (\kappa^*)^2} + \left[ \sigma (1 - \psi^*) + \eta \right] \frac{\sigma + \eta}{\lambda \psi^*} + \frac{\phi_x}{4 \lambda}$$  \hfill (21)$$

For a high worst case belief $\psi^*$, i.e. $\psi^*$ close to two, the product on the right hand side is, given the parameterization outlined above, approximately zero, such that (21) will

\[11\]This is consistent with Söderström’s (2002) result. He finds that uncertainty about the persistence properties of inflation, in the absence of a cost channel, gives rise to more aggressive policy, while uncertainty about other parameters might dampen the policy response.
be negative. As a result, the strength of the interest rate response falls if the central bank’s worry about a misspecified Phillips curve slope grows and hence $\kappa^*$ increases, i.e. $\phi_\pi (\kappa_h) < \phi_\pi (\kappa)$.

Here we consider a model in which the cost channel generates a policy trade-off in the presence of demand shocks as proposed by Ravenna and Walsh (2006). In a model without demand shocks but with supply shocks entering the Phillips curve, however, all results would remain unchanged, i.e. uncertainty leads to an attenuated policy stance. The only difference would be that in this case the decision maker should underestimate $\kappa$ and base his decision on $\kappa = \kappa_l$ rather than $\kappa = \kappa_h$.

Figure (1) depicts $\psi_\pi$ as a function of $\psi_h$ and $\kappa_h$. Apparently, a stronger concern about a misspecified cost channel dampens interest rate responses to inflation. Likewise, uncertainty about $\kappa$ attenuates policy. This is a crucial difference to Giannoni’s (2002) paper and many other contributions. Uncertainty about the cost channel leads to a less vigorous interest rate adjustment, while these papers show that uncertainty about other parameters (in the absence of a cost channel) generally leads to a more aggressive interest rate setting behavior. In this model, therefore, the Brainard (1967) conservatism principle is obeyed. Uncertainty about the transmission of monetary policy to the economy motivates a cautious policy stance. It remains to investigate whether this model is able to quantitatively match empirically observed interest rate setting behavior summarized in estimated Taylor rule coefficients.

### 3.2 Does uncertainty help to match empirical Taylor rules?

A common finding is the discrepancy between empirically estimated Taylor rule coefficients and optimal coefficients derived from an underlying economic model. As Rudebusch (2001) forcefully argues, empirical Taylor rule coefficients suggest an attenuated interest rate response to current inflation. He concludes (p. 214) that accounting for parameter uncertainty and data uncertainty enables the researcher “to reduce the optimal Taylor rule parameters substantially, but plausible amounts of such uncertainty leave the response coefficients ... above their historical estimates.” In this section we investigate whether uncertainty about the cost channel of monetary transmission helps to reconcile optimal policy parameters with actual policy responses.

To establish a benchmark against which theoretically derived optimal Taylor rule coefficients can be contrasted, we estimate a baseline Taylor rule with quarterly U.S. data.
over a sample period that starts with the end of the Volcker disinflation in 1983:1 and ends in 2005:4. Inflation is measured as the yearly change in the GDP deflator and the output gap is based on Congressional Budget Office estimates of potential output. The interest rate is the Federal Funds Rate. The resulting coefficients are (standard errors in parenthesis)

\[ \phi_\pi = 2.18 \quad (0.248) \quad \phi_x = 0.797 \quad (0.122) \]

The numbers closely correspond to Clarida, Galí, and Gertler’s (2000) estimates. Likewise, Jondeau, Le Bihan, and Gallés (2004) find, as their benchmark estimates without interest rate smoothing, \( \phi_\pi = 2.63 \) and \( \phi_x = 0.71 \). Figure (3) shows that the fitted interest rate path closely matches the observed interest rate behavior. Hence, the Taylor rule provides a very reliable descriptive tool to analyze monetary policy and to replicate central bank behavior. As a rough empirical benchmark, the coefficient on inflation lies around two and the coefficient on output gap stabilization around one.

To investigate whether the equilibrium dynamics derived in the theoretical model can be implemented by a Taylor rule with empirically plausible coefficients, we calculate theoretical Taylor rule parameters given in (19) using the benchmark calibration presented in the previous section. For convenience, the output response coefficient is set in order to roughly correspond to empirical estimates, i.e. \( \phi_x = 1.00 \). Note that the value of \( \phi_x \) has only negligible quantitative impact on \( \phi_\pi \), since the expression \( \sigma (1 - \psi) + \eta \) in (19) is close to zero for large worst-case cost channel coefficients. All results are documented in table (2).

First, consider the case of monetary policy under certainty. The optimal Taylor rule exhibits a very strong response to inflation with \( \phi_\pi = 6.0 \), which stands in sharp contrast to almost all estimated Taylor rules. As noted by Rudebusch (2001), uncertainty about the slope coefficient of the Phillips curve alone cannot reconcile these numbers. A policy based on the worst-case \( \kappa \) = \( \kappa_h \) but the benchmark \( \psi = \psi_h = 1.276 \) exhibits

12The data is obtained from the FRED database at the Federal Reserve Board of St. Louis and the Congressional Budget Office’s website, respectively.

13These authors allow, additionally, for interest rate inertia and \( \phi_x = 2.15 \) and \( \phi_x = 0.93 \). See also Judd and Rudebusch (1998) for a prominent empirical paper. If we allow for the lagged interest rate to enter the Taylor rule, the following estimates are obtained

\[
\begin{align*}
i_t & = \left(1 - 0.90\right) \left[ 2.70 \pi_t + 1.45 x_t - 0.90 \right] + 0.10i_{t-1} \\
& = \left[ 0.741 \pi_t + 0.420 x_t - 1.915 \right] + 0.10i_{t-1}
\end{align*}
\]

Generally, parameter estimates obtained from GMM estimation are virtually identical.
Table 2: Welfare loss under different policy scenarios

<table>
<thead>
<tr>
<th>Policy</th>
<th>Welfare</th>
<th>(\phi_\pi)</th>
<th>(\phi_x)</th>
<th>(\text{var}(\pi_t))</th>
<th>(\text{var}(x_t))</th>
<th>(\mathcal{L})</th>
</tr>
</thead>
<tbody>
<tr>
<td>certainty</td>
<td>(\kappa = 0.15, \psi = 1.276)</td>
<td>6.014</td>
<td>1.00</td>
<td>0.033</td>
<td>0.006</td>
<td>0.035</td>
</tr>
<tr>
<td>non robust policy, distorted model</td>
<td>(\kappa_h = 0.30, \psi_h = 1.90)</td>
<td>2.320</td>
<td>1.00</td>
<td>0.317</td>
<td>0.060</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td>(\kappa_h = 0.30, \psi_h = 1.80)</td>
<td>2.443</td>
<td>1.00</td>
<td>0.277</td>
<td>0.052</td>
<td>0.290</td>
</tr>
<tr>
<td></td>
<td>(\kappa_h = 0.20, \psi_h = 1.80)</td>
<td>3.369</td>
<td>1.00</td>
<td>0.125</td>
<td>0.024</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(\kappa_h = 0.15, \psi_h = 1.80)</td>
<td>4.295</td>
<td>1.00</td>
<td>0.071</td>
<td>0.013</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>(\kappa_h = 0.30, \psi_h = 1.276)</td>
<td>3.402</td>
<td>1.00</td>
<td>0.122</td>
<td>0.023</td>
<td>0.128</td>
</tr>
</tbody>
</table>

| robust policy, distorted model | \(\kappa_h = 0.30, \psi_h = 1.90\) | 1.911 | 1.00 | 0.323 | 0.005 | 0.324 |
| | \(\kappa_h = 0.30, \psi_h = 1.80\) | 2.178 | 1.00 | 0.283 | 0.016 | 0.287 |
| | \(\kappa_h = 0.20, \psi_h = 1.80\) | 2.996 | 1.00 | 0.128 | 0.003 | 0.129 |
| | \(\kappa_h = 0.15, \psi_h = 1.80\) | 3.867 | 1.00 | 0.072 | 0.001 | 0.073 |
| | \(\kappa_h = 0.30, \psi_h = 1.276\) | 4.191 | 1.00 | 0.104 | 0.078 | 0.123 |

A coefficient \(\phi_\pi = 4.2\), which is still far above actually observed Fed policy. The size of the response coefficient to the output gap, \(\phi_x\), has only a negligible impact on the size of the \(\phi_\pi\) coefficient. The table shows that for \(\phi_x = 0\), a coefficient of \(\phi_\pi = 5.90\) is obtained.

Now assume that the policymaker also faces uncertainty about the cost channel coefficient. For \(\kappa_h = 0.30\) and \(\psi_h = 1.80\), that is, with a very modest degree of cost channel uncertainty, we are able to perfectly match the estimated Taylor rule coefficient. The parameter \(\phi_\pi\) falls to 2.18, indicating a substantially more attenuated interest rate policy than under certainty. Note that \(\psi_h = 1.80\) lies well within the range of empirically plausible estimates of Ravenna and Walsh (2006) and Chowdhury et al. (2006). Allowing for a slightly larger, but nevertheless still plausible, degree of
uncertainty, generates a coefficient $\phi_x = 1.91$. Thus, we can formulate a second key result of this paper. A plausible amount of uncertainty about the cost channel of monetary transmission suffices to replicate actual interest rate setting behavior. Since this result is obtained under very plausible degrees of parameter uncertainty, the paper does not seem to suffer from Sims’ (2001) critique, who argues that a minmax decision is often based on economically unreasonable priors.

Second, table (2) also shows that uncertainty entails a welfare loss. However, the policymaker can partly offset this welfare loss by pursuing an optimal robust monetary policy. Take for example a central bank that sets interest rates to minimize the worst-case outcome $\kappa_h = 0.3$ and $\psi_h = 1.80$. The resulting welfare loss is $L_t (f (\vartheta^*), \vartheta^*) = 0.287$. Now consider a central bank that does not take account of uncertainty but sets interest rates as if $\vartheta^* = \vartheta$. The resulting welfare loss would amount to $L_t (f (\vartheta), \vartheta^*) = 0.290$.

Due to the fact that the interest rate rule is restricted to respond only to inflation and output, an indeterminacy problem could arise despite the uniqueness of the allocation given in (15). We derive the determinacy properties in the appendix. As a result, the following condition emerges

$$\Gamma > \phi_x > 1 + \frac{\beta + \kappa_h\psi_h - 1}{\kappa_h (\sigma + \eta)} \frac{1}{4} \phi_x$$

(22)

This condition corresponds to the conventional Taylor principle requiring interest rates to respond more than one-to-one to inflation. Note that, for reasonable parameters, $\Gamma$ is never binding. A cautious central bank sets interest as if $\psi$ equals $\psi_h > \psi$. Hence, the lower bound shifts upward the larger the degree of uncertainty, i.e. the higher $\psi_h$.

Likewise, the lower bound moves upward if $\kappa_h$ becomes larger. Figure (8) depicts the region for determinacy. It turns out that the benchmark Taylor rule derived above guarantees a determinate equilibrium even under uncertainty about the cost channel.

---

14See Sack (2000) for a similar result. He finds that parameter uncertainty can account for the observed gradualist policy stance of the Fed. Cateau (2006) also finds that an aversion to model and data-parameter uncertainty can yield an optimal Taylor rule that matches the empirical Taylor rule. Žaković et al. (2005) derive optimal Taylor rules from a minmax decision problem based on an estimated model of the Euro area. The rules also obey the Brainard principle.
3.3 Uncertainty about the demand side

In this subsection, we examine the implications of uncertainty about the demand-side transmission of policy as represented by the inverse of the real interest rate elasticity of output $\sigma$. The set of uncertain parameters is now given by $\vartheta = (\kappa, \psi, \sigma)$.

Let us assume that the policymaker knows that the true value of $\sigma$ is drawn from the interval $[\sigma_l, \sigma_h]$ with $\sigma_h > \sigma_l > 0$. For a given policy $f^* > 0$, the central banker knows that

$$\frac{\partial L}{\partial \sigma} = 1 + \kappa f^* \eta > 0 \quad (23)$$

Hence, optimal policy under uncertainty overestimates the true $\sigma$ and bases interest rate policy on $\sigma = \sigma_h$. How does uncertainty about the demand side affect the strength of interest rate adjustment? Figure (2) plots $\phi_\pi$ as a function of $\psi_h$ and $\sigma_h \epsilon [0, 3]$, which covers most values found in the literature. Apparently, once the cost channel coefficient crosses a threshold, higher uncertainty about $\sigma$, i.e. a larger $\sigma_h$, dampens the interest rate response to inflation and lends further support to the main results of this paper.

3.4 The role of endogenous policy objectives

A crucial step in recent advances of academic thinking about optimal monetary policy is to acknowledge the endogenous nature of policy objectives. A second order approximation to the representative agent’s utility function generates an objective function for monetary policy that is written in terms of squared deviation of inflation and output from their flexible-price steady-state values. The relative weight attached to these competing objectives depends on the deep parameters of the model. In this sense the objective function for monetary policy is endogenous. Woodford (2003, chapter 6) develops such a policy objective function as an approximation to the utility of the representative household.\footnote{See the appendix for a brief summary.} It turns out that the output weight in the central bank’s objective function is given by

$$\lambda_{end} = \frac{\kappa (\sigma + \eta)}{\theta} \quad (24)$$

where $\theta$ determine the price elasticity of demand for the individual goods. Hence, any variation in $\kappa$ (but not in $\psi$) is reflected in the optimal objective function. Uncertainty
about $\kappa$ translates into uncertainty about $\lambda^{\text{end}}$. If $\kappa < \kappa_h$, then $\lambda^{\text{end}} < \lambda$, i.e. the policymaker should attach a smaller weight to output gap fluctuations. A robustly optimal policy should also consider the impact of the worst case parameter constellation on policy objectives. In particular, this implies that the optimal policy plan $f^*$ is independent of uncertainty about $\kappa$ which cancels out in (11)

$$f^* (\vartheta^*) = \frac{\kappa_h}{\lambda^{\text{end}}} \left[ \sigma (1 - \psi_h) + \eta \right]$$

$$= \frac{\theta}{(\sigma + \eta)} \left[ \sigma (1 - \psi_h) + \eta \right]$$

(25)

Importantly, this dimension of optimal policy, i.e. the effect of parameter uncertainty on the objective of the representative agent’s objective function, has not been taken into account by Giannoni (2002). Only recently, Levin and Williams (2003) and Walsh (2005) treat the endogenous nature of objectives in some detail.

Table (3) reports the resulting Taylor rule coefficients as well as equilibrium variances and the welfare loss when the endogenous nature of policy objectives is properly taken into account. To guarantee consistency with the analysis in the previous section, $\theta$ is set such that under certainty both the endogenous as well as the endogenous $\lambda$ are identical.$^{16}$

<table>
<thead>
<tr>
<th>Policy</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_\pi$</td>
<td>$\phi_x$</td>
</tr>
<tr>
<td>robust policy, distorted model</td>
<td></td>
</tr>
<tr>
<td>$\kappa_h = 0.30, \psi_h = 1.90$</td>
<td>1.832</td>
</tr>
<tr>
<td>$\kappa_h = 0.30, \psi_h = 1.80$</td>
<td>2.015</td>
</tr>
<tr>
<td>$\kappa_h = 0.20, \psi_h = 1.80$</td>
<td>2.941</td>
</tr>
<tr>
<td>$\kappa_h = 0.15, \psi_h = 1.80$</td>
<td>3.867</td>
</tr>
<tr>
<td>$\kappa_h = 0.30, \psi_h = 1.276$</td>
<td>3.402</td>
</tr>
</tbody>
</table>

It turns out that the endogenous nature of objectives gives rise to a lower interest rate response to inflation than for the case of exogenously given weights. Under the

---

$^{16}$Following Woodford (2003), the value used here is equal to $16k/\theta$ rather than $k/\theta$. The reason is that the empirical Taylor rule coefficients pertain to inflation measured as annualized percentage change. The model, on the contrary, is based on quarterly changes.
parameter constellation $\kappa_h = 0.30$ and $\psi_h = 1.80$, the optimal Taylor rule exhibits an inflation response of 2.015, whereas with exogenously given objectives, the coefficient was 2.178. The endogenous nature of policy objectives further attenuates interest rate setting. Furthermore, while the size of the resulting welfare loss is hardly affected, the composition of the welfare loss is. Consistent with the weaker reaction to inflation, the inflation variance is slightly higher than under exogenously given preferences.

4 Conclusions

Although the cost channel of monetary transmission is present in many general equilibrium models of the business cycle, the cost channel coefficient exhibits large variation over time and across countries. As a consequence, the central bank faces uncertainty about the true size of the cost channel. Broadly speaking, uncertainty about the cost channel reflects uncertainty about the role of financial markets in transmitting policy shocks to the supply side of the economy.

This paper derived the consequences of an uncertain supply-side transmission of policy impulses for the design of optimal monetary policy. It was shown that a central bank that performs a minmax approach under uncertainty, i.e. that sets interest rates such as to avoid particularly bad outcomes, sets interest rates less aggressively to curb inflation than under certainty. In this respect, the Brainard (1967) principle of cautious policy in the face of uncertainty continues to hold.

Moreover, the model presented in this paper generates an optimal interest rate rule for policymaking under uncertainty, whose coefficients closely match those obtained from empirical studies of Federal Reserve interest rate policy. Hence, the paper shows that a plausible degree of uncertainty about the true nature of the cost channel suffices to reconcile optimal robust Taylor rule coefficients with actually observed monetary policy decisions. Uncertainty about the cost channel appears to be an important factor in explaining monetary policy. It remains interesting to introduce cost channel uncertainty into a fully specified Dynamic General Equilibrium framework and derive the quantitative implications from Bayesian inference, as proposed by Levin et al. (2005). We leave this issue for future research.
References


21


5 Appendix A: Endogenous policy objectives

Woodford (2003, chapter 6) develops a policy objective function that can be interpreted as an approximation to the utility of the representative household. Assume a continuum of differentiated goods \( y_t (i) \) defined on the interval \([0, 1]\). The composite consumption good is then given by

\[
C_t = \left[ \int_0^1 y_t (i) \frac{\theta - 1}{\theta} \, di \right]^\frac{\theta}{\theta - 1}
\]  
(26)

Each household produced one of these goods and experiences disutility from production. Suppose that labor effort is proportional to output. The period utility of the representative agent is assumed to be

\[
V_t = U (C_t, z_t) - \int_0^1 v (y_t (i), z_t) \, di
\]  
(27)

where \( v (y_t (i), z_t) \) is disutility from producing and \( z_t \) is a vector of exogenous shocks. Woodford then shows that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

\[
E_t \sum_{j=0}^{\infty} \beta^j V_{t+j} \approx \frac{1}{2} \Omega E_t \sum_{j=0}^{\infty} \beta^j \left[ \sigma_t^2 + \frac{\kappa (\sigma + \eta)}{\theta} x_t^2 \right]
\]  
(28)

where \( \Omega \) is a constant (which we set to unity) and \( \lambda^{end} = \frac{\sigma (\sigma + \eta)}{\theta} \). In this expression, \( x_t \) is the gap between output and the output level that would arise under flexible prices conditional on interest rates being constant. Therefore, the size of the cost channel does not affect \( \lambda^{end} \). The steady state level of output is efficient since monopolistic distortions are assumed to be neutralized by appropriate subsidies. This expression clearly shows the cross-equation restriction implied by the underlying theory. Variation in \( \kappa \) should be reflected in variation in the weight \( \lambda \).

6 Appendix B: Determinacy properties

We can substitute the instrument rule (16) into equations (1) and (2) rearrange the resulting system of equations to obtain

\[
\begin{bmatrix}
  x_t \\
  \pi_t
\end{bmatrix} = \frac{1}{\chi} \left[ E_t x_{t+1} \right] + B u_t
\]

25
where \( \chi = \sigma + \frac{1}{4} \phi_x + \phi_x \kappa (\sigma (1 - \psi) + \eta) \) and

\[
A = \begin{bmatrix}
\sigma (1 - \kappa \psi \pi) & 1 - \phi \pi (\beta + \kappa \psi) \\
\kappa \sigma (\sigma + \eta + \psi \frac{1}{4} \phi_x) & \beta \left( \sigma + \frac{1}{4} \phi_x \right) + \kappa (\sigma + \eta) + \kappa \psi \frac{1}{4} \phi_x
\end{bmatrix}
\]

The precise form of \( B \) is irrelevant for the stability analysis. In order to ensure determinacy, we need both of the eigenvalues of \( A \) to be inside the unit circle. The characteristic polynomial is \( X^2 + a_1 X + a_0 \), where

\[
a_1 = -\frac{\sigma - \kappa \sigma \psi \pi + \beta \left( \sigma + \frac{1}{4} \phi_x \right) + \kappa (\sigma + \eta) + \kappa \psi \frac{1}{4} \phi_x}{\sigma + \frac{1}{4} \phi_x + \phi_x \kappa (\sigma (1 - \psi) + \eta)}
\]

\[
a_0 = \frac{\beta \sigma}{\sigma + \frac{1}{4} \phi_x + \phi_x \kappa (\sigma (1 - \psi) + \eta)}
\]

Both eigenvalues are inside the unit circle if and only if both of the following conditions hold (Schur-Cohn criterion)

\[
|a_0| < 1 \quad (29)
\]

\[
|a_1| < 1 + a_0 \quad (30)
\]

Condition (29) implies the inequality \( \sigma (\beta - 1) < \frac{1}{4} \phi_x + \phi_x \kappa (\sigma (1 - \psi) + \eta) \), which is satisfied since \( \beta < 1 \) and \( (\sigma (1 - \psi) + \eta) > 0 \).

For \( \sigma - \kappa \sigma \psi \pi + \beta \left( \sigma + \frac{1}{4} \phi_x \right) + \kappa (\sigma + \eta) + \kappa \psi \frac{1}{4} \phi_x > 0 \), condition (30) implies

\[
\phi \pi > 1 + \frac{\beta - 1 + \kappa \psi}{\kappa (\sigma + \eta)} \frac{1}{4} \phi_x
\]

which corresponds to the Taylor principle for a cost channel economy. The Taylor principle requires interest rates to respond more than one-to-one to inflation.\(^{17}\)

The threshold on the left hand side of (31) moves upward as \( \psi = \psi_h \) increases, i.e., as the central bank becomes more uncertain. Moreover, the threshold shifts upward if \( \kappa_h \) becomes larger. For \( \psi = 0 \), this condition collapses to Woodford’s (2001) version of the Taylor principle for the consensus forward-looking model. Under the benchmark parameterization, i.e. for \( \psi = \psi_h = 1.80 \) and \( \kappa = \kappa_h = 0.3 \), this condition requires \( \phi \pi > 1.22 \).

\(^{17}\)In the long-run, \( \pi_t = E_t \pi_{t+1}, x_t = E_t x_{t+1}, \) and \( i_t = \pi_t \). The model therefore requires that \( x_t = \left( \frac{1 - \beta - \kappa \psi}{\kappa (\sigma + \eta)} \right) \pi_t \). Hence, the second expression on the right hand side of (31) corresponds to the long-run effect of inflation on output in the presence of a cost channel of monetary transmission.
Note that the condition \( \sigma - \kappa \psi \phi_n + \beta (\sigma + \frac{1}{4} \phi_x ) + \kappa (\sigma + \eta ) + \kappa \psi \frac{1}{4} \phi_x > 0 \) imposes an upper bound on the inflation response coefficient. It can be written as

\[
\phi_n < \frac{\frac{1}{4} \phi_x (\kappa \psi + \beta ) + \kappa (\sigma + \eta ) + \sigma (1 + \beta )}{\sigma \kappa \psi}
\]

Under the benchmark parameterization, this threshold equals 5.50.
Figure 1: Optimal inflation response coefficient $\phi_\pi$ of a robust Taylor rule as a function of $k_h$ and $\psi_h$. 

\[ \phi_\pi = \text{function of } k_h \text{ and } \psi_h \]
Figure 2: Optimal inflation response coefficient $\phi_\pi$ of a robust Taylor rule as a function of $\sigma_h$ and $\psi_h$.

Figure 3: Actual Federal Funds rate and Federal Funds rate implied by the estimated Taylor rule (in %)
Figure 4: Interest rate response to demand shock as a function of $k_h$ and $\psi_h$

Figure 5: Inflation response to demand shock as a function of $k_h$ and $\psi_h$
Figure 6: Output gap response to demand shock as a function of $k_h$ and $\psi_h$.

Figure 7: Welfare loss as a function of $k_h$ and $\psi_h$. 

31
Figure 8: Determinacy condition for a simple Taylor rule in a cost channel economy