Robust Monetary Policy in a Small Open Economy

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Abstract

We study how a central bank in a small open economy should conduct monetary policy if it fears that its model is misspecified. Using a New Keynesian model of a small open economy, we solve analytically for the optimal robust policy rule and the equilibrium dynamics, and we separately analyze the consequences of central bank robustness against misspecification concerning the determination of inflation, output, and the exchange rate. We show that an increase in the preference for robustness may make the central bank respond more aggressively or more cautiously to shocks, depending on the type of shock and the source of misspecification. We also demonstrate that the price of being robust to misspecification in the Phillips curve or the output equation comes in the form of higher output and exchange rate variability, whereas robustness against misspecification in the exchange rate equation comes at the cost of higher inflation variability.

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1 Introduction

Good policy design requires a good understanding of private sector behavior. Such an understanding is important not only in order to identify market deficiencies and hence policy objectives, but also when trying to reach objectives in the best possible way. The New Keynesian model as laid out by Rotemberg and Woodford (1997), Goodfriend and King (1997), and others has been established as the mainstream model for monetary policy analysis. This model captures the sluggish adjustment of prices and the intertemporal consumption decision in a model framework with optimizing households and firms. With only a limited number of equations and clear intuition, the model has been very influential and has provided policymakers with several guiding policy principles in responding to the different disturbances in the economy (see, for example, Clarida, Galí and Gertler, 1999; and King, 2000). More recently, the New Keynesian framework has been extended to open economies, for example by Galí and Monacelli (2005) and Clarida, Galí and Gertler, (2001).

Although the New Keynesian model has many attractive theoretical properties, it has been criticized by many researchers, most notably for not fitting the data well (see, for example, Ball, 1994; Mankiw, 2001; or Estrella and Fuhrer, 2002). One response to such criticism is to design more complex models that are better able to capture the behavior of macroeconomic variables, following Christiano, Eichenbaum, and Evans (2005) and others. Such models gain in realism but lose in tractability. An alternative route, explored in this paper, is to acknowledge that the simple model is a misspecified description of reality, and to design policy to be robust against model misspecification. We will assume that the true model of private sector behavior lies in some neighborhood around a reference model, and we analyze how monetary policy should be designed in order to work reasonably well for all models inside this neighborhood. This problem has recently been addressed by Hansen and Sargent (2006) using "robust control" techniques. Assuming that the policymaker is unable to formulate a probability distribution over plausible models, the robust policymaker designs policy for the worst possible outcome within a pre-specified set of models.

We apply robust control techniques developed by Hansen and Sargent (2006) and Giordani and Söderlind (2004) to a simple New Keynesian open-economy model developed by Galí and Monacelli (2005) and Clarida et al. (2001). The simple model structure allows us to find closed-form solutions for the optimal robust policy and the equilibrium behavior of the economy. We also generalize the standard robust control framework by allowing the policymaker's preference for robustness to differ across equations, reflecting the confidence the policymaker has in each relationship. For instance, the policymaker may be quite confident about one of the equations (such as the Phillips curve) and believe that robustness against misspecification in this equation is not very important, but at the same time be very uncertain about some other equation (for example, the exchange rate relationship). Our approach allows us to consider each equation in turn and ask what is the appropriate response of robust policy to misspecification in one particular equation. Thus we will consider several different types of misspecification within the model: misspecification in firms' price-setting, misspecification in consumer behavior, and misspecification in the model determining the exchange rate.[1]

The ability to focus on specification errors in particular equations seems important. Policymakers are more confident in some relationships than in others, and so regard some types of specification errors to be more important than others. In open economies, monetary policymakers are particularly uncertain about the effects of the exchange rate on the economy and the effects of monetary policy on the exchange rate. Using our approach, we are able to analyze the design of monetary policy under such specific model uncertainty while keeping other potential sources of misspecification fixed.

One important part of the analysis will focus on how the central bank's preference for robustness against model misspecification affects its monetary policy. Thus far, there is no consensus about whether increased uncertainty should lead to more aggressive or more cautious policy behavior. Following the seminal analysis of Brainard (1967), it is well-accepted that increased uncertainty about the effects of policy should lead to more cautious policy behavior, at least within a Bayesian treatment of model uncertainty. However, Craine (1979) and Söderström (2002) show that this result does not generalize to all parameters in the model: increased uncertainty about the persistence of inflation should instead make policy more aggressive.[2]

Within the robust control literature, several authors (for example, Onatski and Stock, 2002; Hansen and Sargent, 2001; Giannoni, 2002; and Giordani and Söderlind, 2004) have shown that an increased preference for robustness tends to lead to more aggressive policy behavior. These authors rely on numerical methods to solve for the optimal robust policy in a closed economy. In a companion paper, Leitemo and

[1]In an earlier paper (Leitemo and Söderström, 2005), we study the effects of exchange rate model misspecification on the performance of optimized simple monetary policy rules. In that framework, the central bank is uncertain about the exchange rate model, but private agents have perfect information about the exact specification of the model. In the robust control approach used in the present paper the central bank fears misspecification and private sector expectations reflect this fear.

[2]See also Kimura and Kurozumi (2007) and Moessner (2005) for similar results within the New Keynesian framework with forward-looking expectations.

Söderström (2004), we use our analytical approach to show that the aggressiveness result is an inherent feature of robust policy in a closed economy.

We show in the present paper that this result does not carry over to the open economy: depending on the source of misspecification and the type of disturbance affecting the economy, the optimal robust policy in an open economy can be either more aggressive or more cautious than the non-robust policy. A second set of results concerns the effects on the macroeconomy of the central bank's fear of model misspecification. As the central bank designs policy to do well in the worst-case scenario, this will have important consequences for the economy in other more likely outcomes. We show that the price of being robust to misspecification in the Phillips curve or the output equation comes in the form of higher output and exchange rate variability, whereas robustness against misspecification in the exchange rate equation comes at the cost of higher inflation variability.

The robust control approach to model uncertainty is designed to minimize the consequences of the worst-case specification of the policymaker's reference model, and thus focuses exclusively on the worst-case outcome within a set of admissible models.[3] In contrast, a central bank following a Bayesian approach would take into account all possible outcomes in the specified model set and assign weights to each competing model according to its prior probability (see, for instance, Brock, Durlauf, and West, 2004; or Batini, Justiniano, Levine, and Pearlman, 2006). One reason to focus on the worst-case specification is that the policymaker may face Knightian uncertainty and therefore be unable to assign probabilities to alternative specifications of its model. Nevertheless, we do not believe one approach to be superior to the other: instead, both approaches capture important elements of policymaking under uncertainty.

The paper is organized as follows. We first present our model of a small open economy in Section 2. We then derive the robust policy and the equilibrium under this policy in Section 3, both for the "worst-case" model where misspecification is present and for the "approximating" model where there is no misspecification but the central bank's reference model is correct. We analyze the effects of a stronger central bank preference for robustness in Section 4, and we illustrate these effects using a numerical example in Section 5. Finally, we summarize our findings and conclude in Section 6.

[3] Küster and Wieland (2005) and Zaković, Rustem, and Wieland (2005) therefore interpret the robust control policy (and minmax policies in general) as insurance policies.

2 A stylized model of a small open economy

We use a stylized New Keynesian model of a small open economy developed by Galí and Monacelli (2005) and Clarida et al. (2001), which is a generalization of the canonical New Keynesian model for a closed economy developed by Rotemberg and Woodford (1997), Goodfriend and King (1997), and others. We deviate from Galí and Monacelli (2005) and Clarida et al. (2001) by introducing a time-varying premium on foreign bond holdings. This enables us to analyze misspecification concerning the model determining the exchange rate, which is an important source of uncertainty in open economies. It also implies that our model is not isomorphic to a closed-economy model, so the policy descriptions will be qualitatively different from those of a closed economy (see below). We here only present the main relationships in the model; a complete derivation is given in Appendix A.

The world is assumed to consist of two countries: a small open home country and a large foreign country. The two countries share preferences and technology and produce traded consumption goods. In the home country, domestic firms produce goods using labor as the only input, and households enjoy consumption of domestically produced and imported goods and dislike supplying labor to firms.

Define by π_t the rate of inflation in the domestic goods sector; by x_t the output gap in the domestic economy, that is, the log deviation of domestic output from its flexible-price level; and by e_t the real exchange rate, defined in terms of the domestic price level as

$$e_t = s_t + p_t^f - p_t, \tag{1}$$

where s_t is the nominal exchange rate, p_t^f is the price level in the foreign economy, and p_t is the price level of domestically produced goods.[4]

The optimizing behavior of households and firms then implies that the domestic inflation rate, the output gap, and the real exchange rate are interrelated according

[4]Formally, e_t defined as in equation (1) is the terms of trade, the difference between the price of imported goods (the foreign price level expressed in domestic currency) and domestically produced goods. A more traditional way of defining the real exchange rate would be in terms of the domestic consumer price index as

$$q_t = s_t + p_t^f - p_t^c,$$

where $p_t^c = (1-\omega)p_t + \omega(p_t^f + s_t)$, and where ω is the share of imports in domestic consumption. However, as it is more convenient to express the model in terms of e_t, we will refer to it as the real exchange rate. And as the traditional real exchange rate q_t is related to e_t by

$$q_t = (1-\omega)e_t,$$

changes in e_t are proportionally reflected in changes in q_t.

to

$$\begin{aligned}
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t - \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, && (2)\\
x_t &= \mathrm{E}_t x_{t+1} - \frac{1}{\sigma}\left[i_t - \mathrm{E}_t \pi_{t+1}\right] - \gamma\left[\mathrm{E}_t e_{t+1} - e_t\right] + \Sigma_x \varepsilon_t^x, && (3)\\
e_t &= \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1}\right] + \Sigma_e \varepsilon_t^e. && (4)
\end{aligned}$$

Equation (2) is a New Keynesian Phillips curve for the open economy, where the rate of domestic inflation depends on expected future inflation and current marginal cost, which is affected by the output gap and the exchange rate. The real exchange rate affects marginal cost through households' labor supply decision: households value their wage relative to the consumer price index (which includes prices of imported goods), so the equilibrium wage depends on the real exchange rate. The inflation shock, ε_t^π, is due to productivity disturbances that affect the flexible-price level of the real exchange rate.

Equation (3) is an expectational IS curve that relates the output gap to the expected future output gap, the real interest rate (as households substitute consumption over time), and the real exchange rate (as consumption is partly satisfied through imported goods). The demand shock ε_t^x reflects productivity disturbances that affect the flexible-price level of output, or, equivalently, changes in the natural real interest rate.

Finally, equation (4) is a real uncovered interest rate parity (UIP) condition, where the expected rate of real depreciation is related to the real interest rate differential (also in terms of domestic inflation) between the domestic and foreign economies. All foreign variables are assumed to be exogenous, and are therefore set to zero. The exchange rate disturbance, ε_t^e, reflects the fact that domestic households pay a premium on foreign bond holdings.

All disturbances ε_t^j are assumed to be white noise, and the Σ_j parameters denote the standard deviation of each shock so the ε_t^j disturbances have unit variance. The assumption of white noise disturbances allows us to find a closed-form solution for the robust control problem.[5]

The derivation in Appendix A gives a structural interpretation to all parameters in the model, following Galí and Monacelli (2005) and Clarida et al. (2001).[6] The

[5]For the same reason we also assume that there is no endogenous persistence in the model, a feature that is at odds with the data. This assumption does not seem very restrictive, however; extensions using numerical methods indicate that our qualitative results typically go through also in specifications with endogenous or exogenous persistence. These results are available upon request.

[6]Galí and Monacelli (2005) and Clarida et al. (2001) show that with perfect international risk sharing (that is, without a premium on foreign exchange) the open-economy model is isomorphic to the closed economy, so all closed-economy results go through qualitatively also in the open

parameter $\beta \in [0,1]$ is the discount factor of domestic households and firms, while κ, α σ, and γ depend on structural parameters according to

$$\kappa \equiv \hat{\kappa}\,\frac{(1-\omega)\eta+\hat{\sigma}}{1-\omega} > 0, \tag{5}$$

$$\alpha \equiv \hat{\kappa}\,\frac{\Omega-(1-\omega)}{1-\omega}, \tag{6}$$

$$\sigma \equiv \frac{\hat{\sigma}}{1-\omega} > 0, \tag{7}$$

$$\gamma \equiv \frac{\Omega-(1-\omega)}{\hat{\sigma}}, \tag{8}$$

where

$$\hat{\kappa} \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} > 0, \tag{9}$$

$$\Omega \equiv (1-\omega)^2+(2-\omega)\omega\delta\hat{\sigma} > 0, \tag{10}$$

and where $\theta \in (0,1)$ is the probability that a firm is not able to change its price in a given period in the staggered price-setting model of Calvo (1983); $\hat{\sigma} > 0$ is the inverse of the elasticity of intertemporal substitution; $\eta > 0$ is the elasticity of the representative household's labor supply; $\omega \in [0,1]$ is the share of imports in domestic consumption, that is, the degree of openness; and $\delta > 0$ is the elasticity of substitution across domestic and foreign goods. Clearly, β, κ, and σ are always positive, while also α and γ are positive for reasonable parameterizations.

Consequently, movements in the exchange rate tend to have a negative effect on inflation, for a given output gap. This is a combination of two opposing effects. On the one hand, an exchange rate depreciation (an increase in the exchange rate) increases consumer prices and therefore reduces the real wage facing households. Given households' marginal rate of substitution between leisure and consumption, the optimal labor supply choice implies that the real product wage facing firms must increase to offset the reduction in the households' real wage. Therefore, marginal cost and inflation increase. On the other hand, an exchange rate depreciation leads to a decrease in the demand for imports and therefore a reduction in aggregate

economy. In the case of imperfect risk sharing, however, this is no longer true as the foreign exchange premium adds a new source of volatility (and misspecification) that is not present in the closed economy. As we are particularly interested in misspecification concerning the UIP condition, we assume that risk sharing is imperfect, so the open economy is not isomorphic to the closed economy. Such a premium is often assumed to be a function of the deviation of the home country's net foreign position from steady state, an assumption that also ensures a well-defined steady state for consumption and asset holdings. As the net foreign asset position is zero in steady state, the premium is also zero in steady state. (See Benigno, 2001, or Schmitt-Grohé and Uribe, 2003, for details.)

consumption, given the level of output. The marginal rate of substitution then falls, leading to lower real wages and marginal cost. As long as $\delta\hat{\sigma} > (1-\omega)/(2-\omega)$, which is typically the case, the second effect is stronger than the first, so the total effect of exchange rate movements on inflation is negative, given the output gap. In what follows, we will assume this to be the case.

There are also two opposing effects of exchange rate movements on the output gap. On the one hand, an exchange rate depreciation increases consumer prices and reduces the expected rate of inflation, given the expected future price level. This leads to an increase in the real interest rate facing households and therefore a decrease in consumption and the output gap, given the expected future exchange rate. On the other hand, the exchange rate depreciation increases export demand and therefore output. Again, if $\delta\hat{\sigma} > (1-\omega)/(2-\omega)$ the second effect is stronger than the first, so the total effect is positive: given the expected exchange rate, an exchange rate depreciation increases the output gap.

3 Robust monetary policy

3.1 Introducing model misspecification

We close the model by assuming that the short-term interest rate i_t is set by a central bank to minimize a standard objective function that is quadratic in deviations of inflation and the output gap from their zero target levels:

$$\min_{\{i_t\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right], \tag{11}$$

where λ is the central bank's weight on output stabilization relative to inflation stabilization.[7] However, the central bank worries about model misspecification: although the model (2)–(4) is seen as the most likely model, the central bank acknowledges that this reference model may be misspecified. Therefore, it wants to design policy to be robust against reasonable deviations from the reference model. To formalize these fears of model misspecification, we follow Hansen and Sargent (2006) and introduce in each equation a specification error, denoted v_t^j. Thus, the misspecified

[7]This objective function is often used to characterize monetary policy with an inflation target, a strategy that is very common in small open economies (see, for example, Svensson, 2000). It is also broadly consistent with most central bank mandates. Galí and Monacelli (2005) show that when $\hat{\sigma} = \delta = 1$ this objective function represents a second-order approximation of the utility loss for the representative consumer resulting from deviations from the optimal strict inflation-targeting policy (in the model without a foreign exchange premium). In our analysis we will keep these objectives constant; although the central bank is aware that the model may be misspecified, it does not take into account that model misspecification may affect its objectives. Walsh (2005) analyzes robust monetary policy when the central bank objectives are endogenous.

model is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t - \alpha e_t + \Sigma_\pi \left[v_t^\pi + \varepsilon_t^\pi \right], \tag{12}$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] - \gamma \left[\mathrm{E}_t e_{t+1} - e_t \right] + \Sigma_x \left[v_t^x + \varepsilon_t^x \right], \tag{13}$$

$$e_t = \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1} \right] + \Sigma_e \left[v_t^e + \varepsilon_t^e \right]. \tag{14}$$

As the central bank is assumed not to be able to assign a probability distribution over alternative deviations from the reference model, it instead designs policy for the worst possible outcome in a neighborhood of the reference model. In this worst-case outcome the specification errors are chosen by a by a fictitious "evil agent" to maximize central bank loss subject to some constraints (to be specified below).

The model where the central bank selects the interest rate to minimize the value of its loss function while the evil agent selects the specification errors to maximize loss will be referred to as the *worst-case model*. This is the outcome that the central bank fears the most and against which it wants monetary policy to be robust. A more likely outcome of the model, on the other hand, is one where the central bank sets policy and agents form expectations to reflect misspecification in the worst-case model, but there is no such misspecification, and the reference model turns out to be correct. Following Hansen and Sargent (2006), we will refer to this model as the *approximating model*.

As is common in the robust control approach, the amount of misspecification measured by v_t^j is scaled by the parameter Σ_j that determines the volatility of the disturbance in equation j. As discussed by Giordani and Söderlind (2004), there can only be model uncertainty if the true model is surrounded by random noise, so if the disturbance in one equation has no variance, then the specification error would be detected immediately. The larger the variance of the disturbance, the larger can the specification error be without being detected.

Finally, although the specification errors enter the model additively, in equilibrium they will feed back from all state variables in the model. They may therefore disturb the model in the same way as multiplicative parameter uncertainty or omitted variables (see Hansen and Sargent, 2006). The robust control formulation of model misspecification thus represents a fairly general approach to model uncertainty.

3.2 Setting up the control problem

To design the robust policy, the central bank takes into account a certain degree of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models. Depending on its preference for robustness, the central bank allocates a budget η_j to the evil agent, that is used to create misspecification in equation j. In contrast to Hansen and Sargent (2006) and Giordani and Söderlind (2004), we will distinguish between different sources of model misspecification by allowing the evil agent to have different budget constraints for the different specification errors. These budget constraints are then given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t (v_t^j)^2 \quad \leq \quad \eta_j, \quad j = \pi, x, e. \tag{15}$$

In a standard non-robust control problem we would have $\eta_j = 0$ for all j, whereas the standard robust control problem would have a common constraint on misspecification in all equations: $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(v_t^\pi)^2 + (v_t^x)^2 + (v_t^e)^2\right] \leq \eta$. In addition to analyzing the general effects of misspecification by letting all η_j be positive, we can therefore also analyze specification errors in one equation at a time by letting one η_j be positive while setting the others to zero. We will demonstrate that robustness against different sources of misspecification can have very different effects on monetary policy.

Following Hansen and Sargent (2006) the robust monetary policy is obtained by solving the minmax problem

$$\min_{\{i_t\}} \max_{\{v_t^j\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda x_t^2\right] \tag{16}$$

subject to the misspecified model (12)–(14) and the evil agent's budget constraints in (15). The central bank thus sets the interest rate to minimize the value of its intertemporal loss function, whereas the evil agent sets its controls to maximize the central bank's loss, given the constraints on misspecification. The Lagrangian for this problem is given by

$$\begin{aligned} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \quad \Big\{ \quad & \pi_t^2 + \lambda x_t^2 - \theta_\pi \left(v_t^\pi\right)^2 - \theta_x \left(v_t^x\right)^2 - \theta_e \left(v_t^e\right)^2 \\ & -\mu_t^\pi \Big[\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t + \alpha e_t - \Sigma_\pi v_t^\pi - \Sigma_\pi \varepsilon_t^\pi\Big] \\ & -\mu_t^x \Big[x_t - \mathrm{E}_t x_{t+1} + \sigma^{-1} \left(i_t - \mathrm{E}_t \pi_{t+1}\right) \\ & \qquad + \gamma \left(\mathrm{E}_t e_{t+1} - e_t\right) - \Sigma_x v_t^x - \Sigma_x \varepsilon_t^x\Big] \\ & -\mu_t^e \Big[e_t - \mathrm{E}_t e_{t+1} + i_t - \mathrm{E}_t \pi_{t+1} - \Sigma_e v_t^e - \Sigma_e \varepsilon_t^e\Big] \Big\}, \end{aligned} \tag{17}$$

where the μ_t^j variables are Lagrange multipliers on the constraints (12)–(14) and the θ_j parameters determine the set of models available to the evil agent against which the policymaker wants to be robust. These parameters are related to the evil agent's budget η_j: as η_j approaches zero, θ_j approaches infinity, and the degree of misspecification approaches zero. (See Hansen and Sargent, 2006.)

Throughout, we will focus on marginal preferences for robustness. If the central bank is sufficiently averse against misspecification, the evil agent is able to destabilize the model so that no stable solution exists. We instead consider modest preferences of robustness so that the worst-case model misspecification is not easily identified by the policymaker.[8] More specifically, we will analyze the effects of small increases in the preference for robustness starting from the non-robust policy, that is, small decreases in each θ_j starting from $\theta_j = \infty$.

3.3 Optimality conditions

We assume that neither the central bank nor the evil agent has access to any commitment mechanism. Consequently, we take expectations as given in the optimization and look for a discretionary equilibrium. From the first-order conditions we can derive the following optimality conditions relating inflation, output and the degree of misspecification to each other:

$$x_t = -\left[\frac{\kappa}{\lambda} - \frac{\alpha}{(\gamma + \sigma^{-1})\lambda}\right] \pi_t \equiv -A\pi_t, \tag{18}$$

$$v_t^\pi = \frac{\Sigma_\pi}{\theta_\pi} \pi_t, \tag{19}$$

$$\begin{aligned} v_t^x &= \frac{\Sigma_x}{\theta_x} \left[\lambda x_t + \kappa \pi_t\right] \\ &= \frac{\alpha \Sigma_x}{(\gamma + \sigma^{-1})\theta_x} \pi_t, \end{aligned} \tag{20}$$

$$\begin{aligned} v_t^e &= -\frac{\Sigma_e}{\sigma \theta_e} \left[\lambda x_t + \kappa \pi_t\right] \\ &= -\frac{\alpha \sigma^{-1} \Sigma_e}{(\gamma + \sigma^{-1})\theta_e} \pi_t, \end{aligned} \tag{21}$$

[8] In numerical approaches to robust control, the preference for robustness can be chosen such that the policymaker cannot distinguish between the approximating and the worst-case models at reasonable levels of statistical significance; see Hansen and Sargent (2006), Giordani and Söderlind (2004), or Dennis, Leitemo, and Söderström (2006).

where[9]

$$A \equiv \frac{\kappa}{\lambda} - \frac{\alpha}{(\gamma + \sigma^{-1})\lambda} > 0. \tag{22}$$

The optimal inflation/output trade-off thus consists of two components. The first component (κ/λ) is equal to that in the closed economy: if monetary policy has stronger effects on inflation through the output gap (κ is large) or if the central bank does not value output stabilization much (λ is small), the optimal trade-off is steeper, so the central bank reduces output more when inflation is high. The second component ($-\alpha/[(\gamma+\sigma^{-1})\lambda]$) is specific to the open economy, where interest movements affect inflation not only through the output gap but also through the exchange rate. As this "exchange rate channel" works in the opposite direction to the traditional demand channel—a higher interest rate leads to an exchange rate appreciation that increases inflation—the central bank in the open economy must balance two opposing effects of monetary policy on inflation. The central bank is therefore more cautious when responding to an increase in inflation if the exchange rate has a stronger effect on inflation (α is large) or if monetary policy has a weaker effect on output, either through the real interest rate (σ is large) or the exchange rate (γ is small). Thus, in these cases the inflation/output trade-off is flatter (A is smaller).

Interestingly, the optimal inflation/output trade-off is not affected by the central bank's preference for robustness. Thus, the central bank's fear of model misspecification will not alter the optimal "targeting rule" in equation (18). This result is closely related to that of Walsh (2004), who shows that the "optimal implicit instrument rule" (similar to the targeting rule) is not affected by central bank robustness against misspecification in a New Keynesian model of a closed economy. However, in both cases the equilibrium dynamics and the optimal (reduced-form) interest rate

[9] To see that $A > 0$, note that

$$\gamma + \sigma^{-1} = \frac{\Omega - (1-\omega)}{\hat{\sigma}} + \frac{1-\omega}{\hat{\sigma}} = \frac{\Omega}{\hat{\sigma}},$$

so

$$\begin{aligned} A &\equiv \frac{\kappa}{\lambda} - \frac{\alpha}{(\gamma + \sigma^{-1})\lambda} \\ &= \frac{\hat{\kappa}}{\lambda}\left[\frac{\eta(1-\omega)+\hat{\sigma}}{1-\omega}\right] - \frac{\hat{\kappa}}{\lambda}\left[\frac{\Omega-(1-\omega)}{1-\omega}\right]\frac{\hat{\sigma}}{\Omega} \\ &= \frac{\hat{\kappa}}{\lambda(1-\omega)}\left[\eta(1-\omega)+\hat{\sigma} - \frac{\hat{\sigma}\left[\Omega-(1-\omega)\right]}{\Omega}\right] \\ &= \frac{\hat{\kappa}}{\lambda\Omega}\left[\eta\Omega + \hat{\sigma}\right] > 0. \end{aligned}$$

rule for the central bank will be affected by the preference for robustness.[10]

We also note that the worst-case specification errors in equations (19)–(21) are larger in absolute value when inflation is far away from steady state, and these errors tend to push inflation even further away, either directly through specification errors in the Phillips curve, or indirectly through a larger output gap or an appreciating exchange rate. Due to the quadratic loss function, the central bank is particularly vulnerable to such additional movements in inflation, as these force the central bank to move the output gap further to achieve the desired trade-off between inflation and the output gap.

Finally, as the specification errors in the misspecified model have been scaled by the standard deviation of the shocks, the degree of misspecification in an equation depends positively on the variance of the shock associated with the equation, given the preference for robustness. As mentioned earlier, this is motivated by the observation that the larger the variance of a given shock, the more difficult it is for the central bank to identify misspecification in that particular equation.

From equations (19)–(21) we see that the Phillips curve is subject to misspecification in most parameterizations of the model. As long as $\Sigma_\pi > 0$ and the central bank wants to be robust (so $\theta_\pi < \infty$), the policymaker will fear misspecification in this equation. Indeed, as discussed in detail in Leitemo and Söderström (2004), in the closed-economy version of the model (when $\alpha = \gamma = 0$), the central bank will only fear misspecification in the inflation equation. In the closed economy, the policymaker is able to counteract any specification errors in the output equation by an appropriate adjustment of the interest rate, so if interest rate movements do not influence central bank loss independently, the central bank does not fear such specification errors. In the open economy, however, the central bank cannot directly offset output shocks by changing the interest rate as this would affect the exchange rate and therefore inflation (see Walsh, 1999). Thus, the existence of an exchange rate channel makes the output equation more prone to misspecification, and the policymaker will fear that output is high when inflation is high, adding to the inflationary pressure.[11]

[10] As shown in Leitemo and Söderström (2004), this result is due to the timing in the game between the central bank and the evil agent. Here we assume that the central bank and the evil agent act optimally given the other player's actions, leading to a Nash equilibrium. If we instead assume that the central bank acts as a Stackelberg leader and takes into account the misspecification of the evil agent when setting the interest rate, the optimal targeting rule will depend on the preference for robustness.

[11] As mentioned above, in the case of perfect risk sharing, as in Galí and Monacelli (2005) and Clarida et al. (2001), the open-economy model is isomorphic to the closed economy so the robust policy is qualitatively the same in the closed and the open economy.

When the effect of the interest rate on output is strong (σ is small), it is easier for monetary policy to offset specification errors in the output equation, so the central bank does not worry much about such output misspecification. On the other hand, the exchange rate equation is more prone to misspecification, as the interest rate movements needed to offset the inflationary effect of these specification errors would have large costly effects on output.

When the exchange rate has a strong effect on inflation (α is large), specification errors in the exchange rate equation are more costly. At the same time it is also more costly for the central bank to offset specification errors in the output equation, as the interest rate movements have a stronger direct effect on inflation that works in the opposite direction. Therefore the central bank worries more about misspecification in the output and exchange rate equations.

Finally, if the exchange rate has a strong impact on output (so γ is large), the central bank will worry less about output and exchange rate specification errors; it is easier to offset the former with monetary policy, whereas the latter have a smaller effect on inflation as output movements offset some of these effects. In the limit, as γ approaches infinity, such specification errors have no consequences for central bank loss.

Our discussion of the optimal inflation/output trade-off and the central bank's worst-case fears for misspecification highlight the crucial differences between monetary policy in open and closed economies. As the central bank in an open economy faces a more difficult trade-off than in the closed economy, it is also more costly to design monetary policy to be robust against model misspecification. The open economy thus both complicates the transmission channels of monetary policy and increases the scope for model misspecification.

3.4 Solving the model

As there is no persistence in the model, the only state variables are the three shocks, $\varepsilon_t^\pi, \varepsilon_t^x$ and ε_t^e, and all expectations are zero. This allows us to find a closed-form solution for the robust control problem. We will thus look for a solution for the endogenous variables π_t, x_t, e_t, the central bank's instrument i_t, and the evil agent's instruments v_t^π, v_t^x, v_t^e in terms of the three shocks. The solution of the *worst-case model* will then be of the form

$$\begin{bmatrix} \pi_t \\ x_t \\ e_t \end{bmatrix} = \begin{bmatrix} a_\pi & a_x & a_e \\ b_\pi & b_x & b_e \\ c_\pi & c_x & c_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}, \tag{23}$$

the worst possible degree of misspecification will be given by

$$\begin{bmatrix} v_t^\pi \\ v_t^x \\ v_t^e \end{bmatrix} = \begin{bmatrix} \hat{a}_\pi & \hat{a}_x & \hat{a}_e \\ \hat{b}_\pi & \hat{b}_x & \hat{b}_e \\ \hat{c}_\pi & \hat{c}_x & \hat{c}_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}, \tag{24}$$

and the policy rule will take the form

$$i_t = d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e. \tag{25}$$

Finally, the approximating model, where policy is conducted according to (25), but there is no misspecification (so $v_t^j = 0$ for all j), will be given by

$$\begin{bmatrix} \pi_t \\ x_t \\ e_t \end{bmatrix} = \begin{bmatrix} \bar{a}_\pi & \bar{a}_x & \bar{a}_e \\ \bar{b}_\pi & \bar{b}_x & \bar{b}_e \\ \bar{c}_\pi & \bar{c}_x & \bar{c}_e \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \\ \varepsilon_t^e \end{bmatrix}. \tag{26}$$

To find these solutions, we begin by looking for the worst-case solution for inflation, output, and the exchange rate in equation (23) and the worst possible degree of misspecification in equation (24). Noting that equations (18)–(21) imply that the output gap and the specfication errors are linearly related to the rate of inflation, we need only to solve for the coefficients in the inflation and exchange rate equations. The next step is to find the optimal policy rule (25), before deriving the solution for the approximating model (26) by using the optimal policy rule in the original model given by (2)–(4).

Note that we allow the evil agent only to respond to the same variables as the policymaker, see equation (24). This differs from the setup of Hansen and Sargent (2006) and Giordani and Söderlind (2004), where the evil agent is also allowed to respond to lagged state variables, thus introducing persistence in the shocks.[12] In our setup, the evil agent is not allowed to introduce serial correlation in the shocks, as there is no such persistence from the outset. This assumption is mainly for tractability, but it is also consistent with a symmetric view about the evil agent's ability to introduce specification errors and the policymaker's ability to counter model misspecification.

[12]This is because Hansen and Sargent (2006) and Giordani and Söderlind (2004) write the model on its state-space form where the shocks are predetermined variables and are written as autoregressive processes without any persistence. The set of state variables then also includes lagged values of the shocks, and the evil agent is allowed to respond to all state variables. Dennis et al. (2006) develop numerical routines that do not require the model to be written on state-space form, so the evil agent is not necessarily able to introduce serial correlation in the shocks. These numerical routines therefore perfectly replicate our analytical results.

3.4.1 The worst-case model and the monetary policy rule

First, to find an expression for the interest rate with the optimal robust policy, we solve the output equation (13) for the interest rate i_t and substitute for x_t and v_t^x using the optimal trade-off in (18) and the worst-case output misspecification in (20). This yields

$$i_t \;=\; (1-\sigma A)\,\mathrm{E}_t\pi_{t+1} + \sigma B\pi_t - \sigma\gamma \mathrm{E}_t\Delta e_{t+1} + \sigma\Sigma_x\varepsilon_t^x, \tag{27}$$

where

$$B \;\equiv\; A + \frac{\alpha\sigma\Sigma_x^2}{(1+\sigma\gamma)\theta_x} > 0. \tag{28}$$

Although equation (27) describes central bank behavior, it is not a true reaction function due to the presence of non-predetermined variables (π_t, e_t, and their expectations) on the right-hand side. Instead, it is an optimal implicit instrument rule, using the terminology of Giannoni and Woodford (2003). In the closed-economy case (when $\alpha = \gamma = 0$) this rule is independent of the preference for robustness, as in Walsh (2004). However, in the open economy this is no longer true as the central bank also fears misspecification in the output equation.[13]

To derive the true policy reaction function in (25) we must first solve for the forward-looking variables π_t and e_t as functions of the underlying shocks. Using the policy trade-off from (18) and the evil agent's control v_t^π from (19) in the Phillips curve (12), we obtain

$$\pi_t \;=\; \beta \mathrm{E}_t\pi_{t+1} - \kappa A\pi_t - \alpha e_t + \frac{\Sigma_\pi^2}{\theta_\pi}\pi_t + \Sigma_\pi\varepsilon_t^\pi, \tag{29}$$

and collecting terms we get

$$C\pi_t = \beta \mathrm{E}_t\pi_{t+1} + \alpha e_t + \Sigma_\pi\varepsilon_t^\pi, \tag{30}$$

where[14]

$$C \;\equiv\; 1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi} > 0. \tag{31}$$

[13] If we introduce a term that penalizes interest rate movements in the central bank's loss function, we are able to derive an optimal implicit instrument rule that is independent of the preference for robustness also in the open economy. Nevertheless, the equilibrium dynamics still depend on the preference for robustness.

[14] Throughout we evaluate the sign of all coefficients when the preference for robustness is small, so each θ_j approaches infinity.

Likewise, using the interest rate from (27) and the expression for v_t^e from (21) in the UIP condition (14) yields

$$(1+\sigma\gamma)e_t \;=\; (1+\sigma\gamma)\mathrm{E}_t e_{t+1} + \sigma A \mathrm{E}_t \pi_{t+1} - D\pi_t - \sigma\Sigma_x \varepsilon_t^x + \Sigma_e \varepsilon_t^e, \tag{32}$$

where

$$\begin{aligned} D &\equiv \sigma B + \frac{\alpha\Sigma_e^2}{(1+\sigma\gamma)\theta_e} \\ &= \sigma A + \frac{\alpha\sigma^2\Sigma_x^2}{(1+\sigma\gamma)\theta_x} + \frac{\alpha\Sigma_e^2}{(1+\sigma\gamma)\theta_e} > 0. \end{aligned} \tag{33}$$

The reduced form for inflation and the exchange rate is of the form

$$\pi_t \;=\; a_\pi \varepsilon_t^\pi + a_x \varepsilon_t^x + a_e \varepsilon_t^e, \tag{34}$$

$$e_t \;=\; c_\pi \varepsilon_t^\pi + c_x \varepsilon_t^x + c_e \varepsilon_t^e, \tag{35}$$

and combining with equations (30) and (32) and applying the methods of undetermined coefficients it is easily shown that the reduced-form coefficients are

$$a_\pi \;=\; \frac{(1+\sigma\gamma)\Sigma_\pi}{E}, \tag{36}$$

$$a_x \;=\; \frac{\alpha\sigma\Sigma_x}{E}, \tag{37}$$

$$a_e \;=\; -\frac{\alpha\Sigma_e}{E}, \tag{38}$$

$$c_\pi \;=\; -\frac{D\Sigma_\pi}{E}, \tag{39}$$

$$c_x \;=\; -\frac{\sigma C\Sigma_x}{E}, \tag{40}$$

$$c_e \;=\; \frac{C\Sigma_e}{E}, \tag{41}$$

where

$$\begin{aligned} E &\equiv (1+\sigma\gamma)C - \alpha D \qquad (42) \\ &= (1+\sigma\gamma)\left[1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi}\right] - \alpha\left[\sigma A + \frac{\alpha\sigma^2\Sigma_x^2}{(1+\sigma\gamma)\theta_x} + \frac{\alpha\Sigma_e^2}{(1+\sigma\gamma)\theta_e}\right] > 0. \end{aligned}$$

(See Appendix B for details.) Thus, a_π, a_x, and c_e are positive while a_e, c_π, and c_x are negative.

Equations (19)–(21) then imply that the coefficients in the central bank's worst-case misspecification are given by

$$\hat{a}_j \;=\; \frac{\Sigma_\pi}{\theta_\pi} a_j, \tag{43}$$

$$
\begin{aligned}
\hat{b}_j &= \frac{\alpha\Sigma_x}{(\gamma+\sigma^{-1})\theta_x} a_j, && (44)\\
\hat{c}_j &= -\frac{\alpha\sigma^{-1}\Sigma_e}{(\gamma+\sigma^{-1})\theta_e} a_j. && (45)
\end{aligned}
$$

Using the solution for inflation and the exchange rate in the interest rate equation (27), the reduced-form solution for the interest rate is

$$
\begin{aligned}
i_t &= \sigma B\pi_t + \sigma\gamma e_t + \sigma\Sigma_x\varepsilon_t^x \\
&= d_\pi\varepsilon_t^\pi + d_x\varepsilon_t^x + d_e\varepsilon_t^e, && (46)
\end{aligned}
$$

where

$$
\begin{aligned}
d_\pi &= \sigma\left[Ba_\pi + \gamma c_\pi\right], && (47)\\
d_x &= \sigma\left[Ba_x + \gamma c_x + \Sigma_x\right], && (48)\\
d_e &= \sigma\left[Ba_e + \gamma c_e\right], && (49)
\end{aligned}
$$

which are all positive (again, see Appendix B for details).

Thus, for a modest preference for robustness, monetary policy responds positively to each disturbance: positive realizations of the inflation, output or exchange rate disturbances all make the central bank raise the interest rate. The result that monetary policy is tightened after positive inflation or output disturbances is well-known from the closed-economy version of the model, see, for example, Clarida et al. (1999): to counter such inflationary impulses the central bank needs to raise the interest rate to create a negative output gap. In the open economy these policy responses also affect output and inflation through the exchange rate: as the central bank tightens policy, the real exchange rate appreciates to allow for expectations of a future depreciation. Although these exchange rate movements add to inflationary pressure through the Phillips curve, they also reduce the output gap further, thus on balance helping the central bank to stabilize the economy.

The central bank also raises the interest rate after a positive exchange rate disturbance. The exchange rate depreciation and the expectation of a future appreciation tend to reduce inflation but increase output. On balance, the optimal policy response is to increase the interest rate to offset the initial depreciation and the effects on inflation and output.

The reduced-form coefficients in equations (36)–(41) show that inflation in the worst-case equilibrium increases after positive shocks to inflation and output, but falls after a positive exchange rate shock. This is what we would expect from the original model, although the effects are magnified by the specification errors (see equation (43)) and partly offset by monetary policy (see equation (46)). The optimal policy response implies that output responds in the opposite direction to inflation—output falls after positive inflation or output shocks but increases after a positive exchange rate shock—although the worst-case specification errors partly make the central bank's job more difficult. The exchange rate, finally, depreciates after an exchange rate shock but appreciates after inflation or output shocks, due to the tightening of monetary policy, although again the central bank fears that its job is more difficult than in the reference model.

3.4.2 The approximating model

The solution for the worst-case model derived so far is the reduced form under the worst possible case of misspecification, so the evil agent chooses the specification errors to be as damaging as possible, and the central bank's policy rule and private agents' expectations reflect this misspecification. However, this is also a very unlikely equilibrium. In contrast, a more likely outcome, the "approximating model," is when the policy rule and agents' expectations reflect the central bank's preference for robustness, but there is no misspecification, so the reference model turns out to be correct. This equilibrium thus illustrates the effects of central bank robustness on the reference model.

As in the worst-case model, all expectations are zero. We find the approximating model by using the optimal robust interest rate rule from equation (46) in the original model (2)–(4).[15] This yields

$$\begin{aligned} \pi_t &= \kappa x_t - \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, && (50) \\ x_t &= -\sigma^{-1} i_t + \gamma e_t + \Sigma_x \varepsilon_t^x, && (51) \\ e_t &= -i_t + \Sigma_e \varepsilon_t^e, && (52) \end{aligned}$$

and we show in Appendix B that the reduced-form coefficients in the approximating model in equation (26) are given by

$$\begin{aligned} \bar{a}_\pi &= \Sigma_\pi + \left[\alpha - \kappa(\gamma + \sigma^{-1})\right] d_\pi > 0, && (53) \\ \bar{a}_x &= \kappa \Sigma_x + \left[\alpha - \kappa(\gamma + \sigma^{-1})\right] d_x > 0, && (54) \\ \bar{a}_e &= (\kappa\gamma - \alpha)\Sigma_e + \left[\alpha - \kappa(\gamma + \sigma^{-1})\right] d_e < 0, && (55) \\ \bar{b}_\pi &= -(\gamma + \sigma^{-1}) d_\pi < 0, && (56) \end{aligned}$$

[15] As policy is implemented using the instrument rule (46), which is optimal only for the misspecified model, we can no longer use the optimal output–inflation trade-off (18) to determine the output gap.

$$
\begin{aligned}
\bar{b}_x &= \Sigma_x - (\gamma + \sigma^{-1}) d_x < 0, &(57)\\
\bar{b}_e &= \gamma \Sigma_e - (\gamma + \sigma^{-1}) d_e > 0, &(58)\\
\bar{c}_\pi &= -d_\pi < 0, &(59)\\
\bar{c}_x &= -d_x < 0, &(60)\\
\bar{c}_e &= \Sigma_e - d_e > 0. &(61)
\end{aligned}
$$

As we focus on marginal preferences for robustness, the signs of all coefficients are the same in the approximating and the worst-case models, as well as in the non-robust version of the model. However, the effects of a stronger preference for robustness may well differ between the worst-case and approximating models, also when θ_j is very large. We next analyze analyze how such an increase in central bank robustness affects the optimal design of monetary policy and the economy.

4 The effects of central bank robustness

The main focus of our analysis concerns the effects of the central bank's preference for robustness on the optimal monetary policy and the resulting behavior of the economy. In this Section we therefore analyze the effects on the equilibrium of an increase in the preference for robustness, that is, a decrease in each θ_j. For instance, for the coefficient of inflation on the inflation disturbance in the worst-case model, we will evaluate the derivative

$$
-\frac{\partial |a_\pi|}{\partial \theta_j}, \quad j = \pi, x, e, \tag{62}
$$

which measures the marginal effects on the absolute value of the coefficient a_π of a decrease in each θ_j, holding constant the other θ_i, $i \neq j$. Throughout we consider modest preferences of robustness so that the worst-case model misspecification is not easily identified by the policymaker, that is, we analyze the effects of small decreases in each θ_j starting from $\theta_j = \infty$.

We will first show that the robust central bank always fears that inflation, output, and the exchange rate are more volatile than in the reference model. We then analyze the effects of robustness on the optimal policy, and demonstrate that the robust policy responds more or less aggressively to shocks depending on the shock and the type model misspecification that the central bank fears. Finally, we show how a stronger preference for robustness affects the macroeconomy in the approximating model.

4.1 The worst-case model

To understand the effects of robustness on monetary policy we first study the worst-case model for inflation, output, and the exchange rate. This equilibrium is interesting mainly because it illustrates the central bank's worst fears of misspecification and therefore helps us to understand the design of the robust monetary policy.

Our first result is that the robust central bank always fears that inflation, output, and the exchange rate are more volatile than in the reference model:

Proposition 1 *In the worst-case model, a stronger preference for robustness (against misspecification in any equation) increases the sensitivity of inflation, output, and the exchange rate to all shocks.*

Proof To establish this result for inflation and output, recall that the reduced-form coefficients for inflation in the worst-case model are given by

$$a_\pi = \frac{(1+\sigma\gamma)\Sigma_\pi}{E}, \tag{63}$$

$$a_x = \frac{\alpha\sigma\Sigma_x}{E}, \tag{64}$$

$$a_e = -\frac{\alpha\Sigma_e}{E}, \tag{65}$$

and the output coefficients are given by $b_j = -Aa_j$ for all j. Thus, all inflation and output coefficients depend negatively (in absolute value) on E. As E increases in all θ_j (see equation (42)), the inflation and output coefficients all increase in absolute value when the preference for robustness against misspecification in any equation increases (any θ_j falls). To demonstrate the result for the exchange rate, we need to use some more algebra. The exchange-rate coefficients in the worst-case model are given by

$$c_\pi = -\frac{D\Sigma_\pi}{E}, \tag{66}$$

$$c_x = -\frac{\sigma C\Sigma_x}{E} = \frac{\sigma\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}Ca_\pi, \tag{67}$$

$$c_e = \frac{C\Sigma_e}{E} = -\frac{\Sigma_e}{\sigma\Sigma_x}c_x. \tag{68}$$

First, as D does not depend on θ_π, but decreases in both θ_x and θ_e (see equation (33)) whereas E increases in all θ_j, it follows that c_π decreases in all θ_j in absolute value, and so increases with the preference for robustness. Second, as C does not depend on θ_x and θ_e (see equation (31)), c_x also increases in absolute value with the preference for robustness against output and exchange rate misspecification, whereas for an increase in robustness against inflation misspecification, we obtain

$$
\begin{aligned}
-\frac{\partial|c_x|}{\partial\theta_\pi} &= -\frac{\sigma\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}\left[a_\pi\frac{\partial C}{\partial\theta_\pi}+C\frac{\partial a_\pi}{\partial\theta_\pi}\right] \\
&= -\left[E-(1+\sigma\gamma)C\right]\frac{\sigma\Sigma_x\Sigma_\pi^2}{E^2\theta_\pi^2} \\
&= \alpha D\frac{\sigma\Sigma_x\Sigma_\pi^2}{E^2\theta_\pi^2} > 0. \qquad (69)
\end{aligned}
$$

Finally, the effects of robustness on the absolute value of c_e are the same as those on c_x. □

Thus, with a stronger preference for robustness, the central bank fears that all variables are more sensitive to shocks, and therefore more volatile.[16] While this result is intuitive, as more volatility increases central bank loss, it stands in contrast to the closed-economy case. Leitemo and Söderström (2004) show that the robust central bank in a closed economy fears only that inflation is more volatile than in the reference model, not that output is more volatile. As discussed earlier, this is because output disturbances do not create a trade-off for monetary policy in the closed economy if interest rate movements do not independently affect central bank loss. In the open economy, on the other hand, all disturbances create trade-offs for monetary policy, and the robust central bank therefore fears that all variables are more volatile than in the reference model.

4.2 Monetary policy

We now turn to the effects of central bank robustness on the optimal monetary policy rule given by

$$
i_t = d_\pi\varepsilon_t^\pi + d_x\varepsilon_t^x + d_e\varepsilon_t^e, \qquad (70)
$$

where we recall that all d_j coefficients are positive, see Appendix B.

We then demonstrate the following result:

Proposition 2 *In general, monetary policy may respond more or less aggressively to shocks when the preference for robustness increases, depending on the source of uncertainty and the type of shock. A stronger preference for robustness against inflation and output misspecification makes monetary policy respond more aggressively*

[16] As we do not allow for shock persistence, the central bank fears only that shocks have a larger direct impact, not that they are more persistent (as in Giordani and Söderlind, 2004, or Dennis, Leitemo, and Söderström, 2007).

to inflation and output shocks, but less aggressively to exchange rate shocks, whereas a stronger preference for robustness against exchange rate misspecification has the opposite effects.

Proof It is useful to first write the policy rule coefficients in (47)–(49) as

$$\begin{aligned} d_\pi &= F_\pi a_\pi, & (71)\\ d_x &= F_x a_\pi + \sigma\Sigma_x, & (72)\\ d_e &= F_e a_\pi, & (73) \end{aligned}$$

where

$$\begin{aligned} F_\pi &\equiv \frac{\sigma}{1+\sigma\gamma}B - \frac{\alpha\sigma\gamma\Sigma_e^2}{(1+\sigma\gamma)^2\theta_e} > 0, & (74)\\ F_x &\equiv [\alpha B - \gamma C]\frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi} < 0, & (75)\\ F_e &\equiv [\gamma C - \alpha B]\frac{\sigma\Sigma_e}{(1+\sigma\gamma)\Sigma_\pi} \\ &= -\frac{\Sigma_e}{\sigma\Sigma_x}F_x > 0, & (76) \end{aligned}$$

We showed above that a_π increases with the preference for robustness against misspecification in any equation, and so decreases with all θ_j. It is also easy to establish that F_π is independent of θ_π, decreases with θ_x (as B decreases with θ_x, see equation (28)), and increases with θ_e; F_x decreases with both θ_π and θ_x (as C increases with θ_π), but is independent of θ_e; whereas the derivatives of F_e have the opposite signs to those of F_x.

It then immediately follows that d_π increases with robustness against inflation and output misspecification, d_x decreases with robustness against exchange rate misspecification, and the effects on d_e have the opposite sign to those on d_x. For the remaining derivatives, we obtain

$$\begin{aligned} -\frac{\partial|d_\pi|}{\partial\theta_e} &= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E}\frac{\alpha\sigma\gamma\Sigma_e^2}{(1+\sigma\gamma)^2\theta_e^2} + \left[\frac{\sigma}{1+\sigma\gamma}B - \frac{\alpha\sigma\gamma\Sigma_e^2}{(1+\sigma\gamma)^2\theta_e}\right]\frac{\alpha^2\Sigma_\pi\Sigma_e^2}{E^2\theta_e^2} \\ &= -\left[\sigma\gamma E - \alpha\sigma B + \frac{\alpha^2\sigma\gamma\Sigma_e^2}{(1+\sigma\gamma)\theta_e}\right]\frac{\alpha\Sigma_\pi\Sigma_e^2}{(1+\sigma\gamma)E^2\theta_e^2} \\ &= -\left[\sigma(1+\sigma\gamma)\left(\gamma C - \alpha B\right)\right]\frac{\alpha\Sigma_\pi\Sigma_e^2}{(1+\sigma\gamma)E^2\theta_e^2} < 0, & (77)\\ -\frac{\partial|d_x|}{\partial\theta_\pi} &= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E}\frac{\gamma\sigma^2\Sigma_\pi\Sigma_x}{(1+\sigma\gamma)\theta_\pi^2} - [\alpha B - \gamma C]\frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}\frac{(1+\sigma\gamma)^2\Sigma_\pi^3}{E^2\theta_\pi^2} \\ &= -\frac{\gamma\sigma^2\Sigma_\pi^2\Sigma_x}{E\theta_\pi^2} - [\alpha B - \gamma C]\frac{(1+\sigma\gamma)\sigma^2\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2} \end{aligned}$$

$$
\begin{aligned}
&= -\left[\gamma\left(E+\alpha\sigma B-\sigma\gamma C\right)+\alpha B-\gamma C\right]\frac{\sigma^2\Sigma_\pi^2\Sigma_x}{E^2\theta_\pi^2}>0, && (78)\\
-\frac{\partial|d_x|}{\partial\theta_x} &= -\frac{(1+\sigma\gamma)\Sigma_\pi}{E}\frac{\alpha^2\sigma^2\Sigma_x^2\Theta_x^{-1}}{(1+\sigma\gamma)\Sigma_\pi\theta_x}-\left[\alpha B-\gamma C\right]\frac{\sigma^2\Sigma_x}{(1+\sigma\gamma)\Sigma_\pi}\frac{\alpha^2\sigma^2\Sigma_\pi\Sigma_x^2}{E^2\theta_x^2}\\
&= -\frac{\alpha^2\sigma^3\Sigma_x^3}{(1+\sigma\gamma)E\theta_x^2}-\left[\alpha B-\gamma C\right]\frac{\alpha^2\sigma^4\Sigma_x^3}{(1+\sigma\gamma)E^2\theta_x^2}\\
&= -\left[E+\alpha\sigma B-\sigma\gamma C\right]\frac{\alpha^2\sigma^3\Sigma_x^3}{(1+\sigma\gamma)E^2\theta_x^2}>0, && (79)
\end{aligned}
$$

where we used

$$
E+\alpha\sigma B-\sigma\gamma C=C-\frac{\alpha^2\Sigma_e^2}{(1+\sigma\gamma)\theta_e}>0, \tag{80}
$$

see Appendix B. □

It is intuitive that monetary policy responds more aggressively to inflation and output shocks when it fears that these equations are misspecified, as the worst-case misspecification increases the volatility of all variables. As the exchange rate shock reduces inflation but increases output, and the central bank fears that this shock will reduce inflation more (if the inflation equation is misspecified) or increase output less (if the output equation is misspecified), robustness against inflation or output misspecification makes policy respond less aggressively to the exchange rate shock in order to dampen the effects of misspecification on the economy.

When the central bank fears that the exchange rate equation is misspecified, it responds more aggressively to exchange rate shocks to dampen its effects on inflation and output. It also responds less aggressively to inflation and output shocks. The central bank fears that positive inflation shocks are associated with an exchange rate appreciation that further increases inflation. As a policy tightening to some extent increases inflation even further, the robust policy responds less aggressively in order to reduce the impact on inflation. The bank also fears that positive output shocks are associated with an exchange rate depreciation that increases output but reduces inflation. It therefore chooses to increase the interest rate less aggressively.

Thus, in general there is an ambiguous effect of an increase in the preference for robustness on the optimal monetary policy. Again, this ambiguity is in contrast to the closed-economy case, where a stronger preference for robustness always makes the central bank respond more aggressively to shocks, see Leitemo and Söderström (2004). As the central bank in a closed economy only fears misspecification in the inflation equation, Proposition 2 implies that such fears should make the central bank respond more aggressively to inflation and output shocks. The effects in the open economy are instead ambiguous and depend on the source of uncertainty and the type of shock.

4.3 The approximating model

Finally, we analyze the effects on the approximating model (the more likely outcome) of an increase in the central bank's preference for robustness. As there is no misspecification in the approximating model, the effects of robustness originate exclusively in the robust policy. It is then fairly straightforward to demonstrate the following result:

Proposition 3 *In the approximating model, a stronger preference for robustness against inflation and output misspecification reduces the sensitivity of inflation but increases the sensitivity of output and the exchange rate to all shocks, whereas a stronger preference for robustness against exchange rate misspecification has the opposite effects.*

Proof To evaluate the effects of robustness on the approximating model, note that the approximating model coefficients in equations (53)–(61) imply that

$$\frac{\partial \bar{a}_i}{\partial \theta_j} = \left[\alpha - \kappa(\gamma + \sigma^{-1})\right] \frac{\partial d_i}{\partial \theta_j}, \tag{81}$$

$$\frac{\partial \bar{b}_i}{\partial \theta_j} = -(\gamma + \sigma^{-1}) \frac{\partial d_i}{\partial \theta_j}, \tag{82}$$

$$\frac{\partial \bar{c}_i}{\partial \theta_j} = -\frac{\partial d_i}{\partial \theta_j}, \tag{83}$$

where

$$\begin{aligned} \alpha - \kappa(\gamma + \sigma^{-1}) &= \alpha - \frac{\kappa \Omega}{\hat{\sigma}} \\ &= -\hat{\kappa}\hat{\sigma}(\hat{\sigma} + \eta\Omega) < 0. \end{aligned} \tag{84}$$

As all policy rule coefficients are positive, those approximating model coefficients that are negative will have the same qualitative effects of robustness as the policy coefficients, whereas the positive coefficients will be affected in the opposite direction compared with the policy coefficients.

The inflation coefficients on the inflation and output shocks are both positive; increased robustness therefore has the opposite effects on these coefficients compared with the coefficients in the policy rule. The inflation coefficient on the exchange rate shock is instead negative, so the effects on this coefficient are qualitatively the same as on the coefficient in the policy rule. The output and exchange rate coefficients all have the opposite sign to the inflation coefficients. Thus, in the approximating model, a stronger preference for robustness against inflation and output misspecification reduces the sensitivity of inflation but increases the sensitivity of output and the exchange rate to all shocks. A stronger preference for robustness against exchange rate misspecification, on the other hand, increases the sensitivity of inflation but reduces the sensitivity of output and the exchange rate to all shocks. □

If the central bank fears misspecification in the inflation or output equations, it will respond more aggressively to inflation and output shocks, but less aggressively to exchange rate shocks. This makes inflation less volatile, while output and the exchange rate become more volatile. If instead the central bank fears misspecification in the exchange rate equation, the central bank responds less aggressively to inflation and output shocks but more aggressively to exchange rate shocks, that makes inflation more volatile but output and the exchange rate less volatile.

As the effects of robustness on monetary policy in the open economy depend on the source of uncertainty and the type of shock, so do the effects on the approximating model. In the closed economy, as the robust policy responds more aggressively to shocks, inflation is less volatile and output is more volatile than in the reference model. In the open economy, in contrast, the effects of robustness are ambiguous. Again, the presence of the exchange rate complicates the trade-offs facing the central bank and increases the scope for misspecification. Therefore, the robust monetary policy is considerably more complicated than in a closed economy.

4.4 Summary

Table 1 summarizes the effects of increased robustness on the reduced-form coefficients. The third column shows the sign of each coefficient, and the next three columns show the effects of an increase in the preference for robustness (a decrease in each θ_j) on the absolute values of the reduced-form coefficients. A positive sign implies that the variable in question becomes more sensitive to that particular shock when robustness increases, and vice versa. It is again clear that the effects of robustness on monetary policy are ambiguous: A robust policymaker may respond more or less aggressively to shocks than a non-robust policymaker, depending on both the shock and the source of misspecification.

5 A numerical example

Finally, to obtain a feeling for the quantitative effects of a stronger preference for robustness, this section presents a simple numerical example. Of course, as the model is highly stylized, all quantitative results need to be interpreted with care.

Table 1: Effects of increased robustness on reduced-form coefficients

Equation	Coefficient on	Sign	Source of misspecification		
			Inflation (θ_π)	Output (θ_x)	Exchange rate (θ_e)
Worst-case model					
Inflation (π_t)	Inflation (a_π)	+	+	+	+
	Output (a_x)	+	+	+	+
	Exchange rate (a_e)	−	+	+	+
Output (x_t)	Inflation (b_π)	−	+	+	+
	Output (b_x)	−	+	+	+
	Exchange rate (b_e)	+	+	+	+
Exchange rate (e_t)	Inflation (c_π)	−	+	+	+
	Output (c_x)	−	+	+	+
	Exchange rate (c_e)	+	+	+	+
Policy rule					
Interest rate (i_t)	Inflation (d_π)	+	+	+	−
	Output (d_x)	+	+	+	−
	Exchange rate (d_e)	+	−	−	+
Approximating model					
Inflation (π_t)	Inflation ($\bar{a}_\pi$)	+	−	−	+
	Output ($\bar{a}_x$)	+	−	−	+
	Exchange rate ($\bar{a}_e$)	−	−	−	+
Output (x_t)	Inflation ($\bar{b}_\pi$)	−	+	+	−
	Output ($\bar{b}_x$)	−	+	+	−
	Exchange rate ($\bar{b}_e$)	+	+	+	−
Exchange rate (e_t)	Inflation ($\bar{c}_\pi$)	−	+	+	−
	Output ($\bar{c}_x$)	−	+	+	−
	Exchange rate ($\bar{c}_e$)	+	+	+	−

For each coefficient in the reduced-form model, Column 3 shows the sign of the coefficient, and Columns 4–6 show the effects of an increased central bank preference for robustness (a lower θ_j) on the absolute value of the coefficient. Thus, +/− implies that increased robustness makes the variable in question more/less sensitive to that particular shock.

Nevertheless, this example will illustrate the relative importance of the different sources of misspecification on the model coefficients.

To parameterize the model, we take values for the structural parameters from Galí and Monacelli (2005): $\hat{\sigma} = \delta = 1$, $\eta = 3$, $\theta = 0.75$, $\beta = 0.99$, and $\omega = 0.4$. This implies that the coefficients in the model (2)–(4) are given by $\kappa = 0.401$, $\alpha = 0.057$, $\sigma = 1.667$, and $\gamma = 0.4$. Finally, we set the relative weight on output stabilization in the central bank's loss function to $\lambda = 0.25$, and the shock variances Σ_j are all set to unity.[17]

We then investigate how an increase in the preference for robustness against one source of misspecification (that is, a decrease in each θ_j, keeping the other θ's fixed at a large value) affects the parameters in the central bank's worst-case model, the policy rule and the approximating model. The results are reported in Figures 1–7.[18] It is immediately clear that a stronger preference for robustness (moving from right to left in each panel) has different quantitative effects on the coefficients in the worst-case and approximating models as well as in the policy rule. In general, there are large effects of all sorts of misspecification fears on the coefficient on inflation shocks in all equations, both in the worst-case model, the approximating model and in the policy rule, whereas the effects are substantially smaller for most other coefficients. This reflects the fact that inflation shocks pose the most difficult trade-off for the central bank, as there are no direct effects of monetary policy on inflation, only through the output gap and the exchange rate.

Figures 8–10 instead show how the volatility in each variable is affected by the central bank's preference for robustness. In the worst-case model, Figure 8 confirms our finding that as the central bank's preference for robustness increases it fears that all variables are more volatile than in the reference model. The effects tend to be more pronounced for the exchange rate and output than for inflation, which is fairly stable, as the central bank attaches a large weight to inflation stability. The effects on interest rate volatility are in general ambiguous, but Figure 9 shows that with our calibration interest rate volatility is larger when the central bank fears misspecification in the inflation of output equations, but smaller when the central bank fears misspecification in the exchange rate equation. This is because the strong effects on the inflation and output coefficients tend to dominate the opposing effects

[17] In the objective function derived as a second-order approximation to utility, Galí and Monacelli (2005) show that $\lambda = (1-\theta)(1-\beta\theta)(1+\eta)/(\epsilon\theta)$, where ϵ is the elasticity of substitution across the differentiated domestic goods. Using their value of $\epsilon = 6$, this implies that $\lambda = 0.0572$. We use a slightly larger (and possibly more realistic) value for λ. However, the qualitative results are not sensitive to the value of λ.

[18] The upper bounds on the θ's in the figures are sufficiently large to ensure that the solution is essentially identical to the non-robust case.

on the exchange rate coefficient. The effects on volatility in the approximating model, on the other hand, are unambiguous, and Figure 10 shows that as in the worst-case model, the effects on output and exchange rate volatility are stronger than on inflation volatility.

6 Concluding remarks

We use a stylized model of a small open economy to analyze how optimal monetary policy and the behavior of the economy are affected by the central bank's desire to be robust against model misspecification. The simple model structure enables us to solve analytically for the optimal robust policy, as well as the central bank's worst-case model and the more likely approximating model. Our framework also allows us to analyze cases when the policymaker is more confident about some equations in the model than others.

Our analysis shows that an increase in the central bank's preference for robustness has ambiguous effects on the optimal policy behavior, depending not only on the shock to which the central bank responds, but also on what part of the model the central bank perceives as most prone to misspecification. Although our model is highly stylized, we believe this ambiguity to carry over also to more elaborate models.[19] In numerical applications the effects of increased misspecification will therefore depend crucially on the calibration or estimation of the parameters that determine the central bank's relative faith in the different model equations.

In a companion paper (Leitemo and Söderström, 2004) we focus on the optimal robust policy in the closed-economy version of our model. There, the results are unambiguous: the robust policy always responds more aggressively to shocks than the non-robust policy, and, as a consequence, inflation is less volatile and output is more volatile under the robust policy. The present paper shows that the effects of robustness in the open economy are more complex, as is the design of monetary policy in general. This is because in an open economy the presence of the exchange rate produces new trade-offs for monetary policy and introduces an additional source of volatility and model misspecification.

[19] This is also confirmed by Dennis, Leitemo, and Söderström (2007) who study robust monetary policy in larger open-economy model estimated on data from the U.K.

A Model appendix

This Appendix derives our open-economy model from microfoundations following Galí and Monacelli (2005) and Clarida et al. (2001). We deviate from these authors by introducing a time-varying premium on foreign exchange in order to analyze uncertainty about exchange rate determination.

A.1 Domestic households

Households in the home country consume a CES composite of domestically produced goods (C_t^d) and imported foreign goods (C_t^m), defined as

$$C_t = \left[(1-\omega)^{1/\delta}(C_t^d)^{(\delta-1)/\delta} + \omega^{1/\delta}(C_t^m)^{(\delta-1)/\delta}\right]^{\delta/(\delta-1)}, \tag{A1}$$

where ω is the share of foreign goods in consumption and δ is the elasticity of substitution across domestic and foreign goods. Households obtain utility from consumption and disutility from supplying labor (N_t) according to

$$U(C_t, N_t) = \frac{C_t^{1-\hat{\sigma}}}{1-\hat{\sigma}} - \frac{N_t^{1+\eta}}{1+\eta}, \tag{A2}$$

where $\hat{\sigma}$ is the inverse of the elasticity of intertemporal substitution and η is the elasticity of labor supply.

The household chooses paths of consumption, labor supply, and holdings of one-period domestic bonds, which pay the nominal interest rate i_t, and foreign bonds, which pay the risk-adjusted interest rate $\exp(\phi_t)i_t^f$, where ϕ_t is a time-varying premium on foreign bond holdings and i_t^f is the one-period nominal interest rate in the foreign economy. Intertemporal optimization then gives the log-linearized consumption Euler condition[20]

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\hat{\sigma}}\left[i_t - \mathrm{E}_t \pi_{t+1}^c\right], \tag{A3}$$

where $\pi_t^c \equiv p_t^c - p_{t-1}^c$ is the inflation rate in terms of the consumer price index (CPI)

$$p_t^c = (1-\omega)p_t + \omega p_t^m, \tag{A4}$$

where p_t and p_t^m are the price levels for domestically produced and imported goods, respectively.

The optimal allocation across domestic and foreign bond holdings gives the uncovered interest rate parity (UIP) condition

$$i_t = i_t^f + \mathrm{E}_t \Delta s_{t+1} + \phi_t, \tag{A5}$$

[20]Throughout, lower-case letters denote log deviations from steady state.

where s_t is the nominal exchange rate and ϕ_t is the premium on foreign exchange. The optimal labor-leisure choice implies that

$$\eta n_t + \hat{\sigma} c_t = w_t - p_t^c. \tag{A6}$$

where w_t is the nominal wage. Finally, relative demand for domestic and imported goods satisfies

$$c_t^d - c_t^m = -\delta \left[p_t - p_t^m \right]. \tag{A7}$$

We define the real exchange rate in terms of the domestic price level as

$$e_t = s_t + p_t^f - p_t, \tag{A8}$$

which, assuming that the law of one price holds, is equal to the terms of trade $p_t^m - p_t$. Then we can express the UIP condition (A5) in real terms as

$$i_t - \mathrm{E}_t \pi_{t+1} = i_t^f - \mathrm{E}_t \pi_{t+1}^f + \mathrm{E}_t \Delta e_{t+1} + \phi_t. \tag{A9}$$

We can also write the CPI in (A4) as

$$p_t^c = p_t + \omega e_t, \tag{A10}$$

the CPI inflation rate as

$$\pi_t^c = \pi_t + \omega \Delta e_t, \tag{A11}$$

and the labor supply condition in (A6) as

$$\eta n_t + \hat{\sigma} c_t = w_t - p_t - \omega e_t. \tag{A12}$$

Log-linearizing the consumption index (A1), we get

$$c_t = (1 - \omega) c_t^d + \omega c_t^m, \tag{A13}$$

and combining with (A7) and (A8) to eliminate c_t^m gives

$$c_t = c_t^d - \omega \delta e_t. \tag{A14}$$

A.2 Domestic firms

Domestic firms act under monopolistic competition and produce a differentiated good using only labor inputs according to the production function

$$Y_t = \exp(a_t)N_t, \tag{A15}$$

where a_t is a productivity disturbance.

Firms face a constant elasticity demand curve for its output, and set their prices in a staggered fashion following Calvo (1983), so in each period there is a fixed probability $1-\theta$ that the firm will be able to change its price. When prices can be adjusted, firms maximize the expected discounted value of profits. This implies that inflation in the domestic sector follows the New-Keynesian Phillips curve

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \hat{\kappa} v_t, \tag{A16}$$

where β is a discount factor, $\hat{\kappa} \equiv (1-\theta)(1-\beta\theta)/\theta$ and v_t is real marginal cost, given by

$$v_t = w_t - p_t - a_t, \tag{A17}$$

where $w_t - p_t$ is the real product wage, deflated by the domestic price level.

A.3 The foreign economy

The foreign economy is assumed to be large relative to the domestic economy, so domestic variables have no impact on the foreign economy. For simplicity, we assume that preferences and technology are identical across the two economies. Foreign demand for domestic goods is then given by

$$c_t^{df} = y_t^f + \delta e_t, \tag{A18}$$

where y_t^f is foreign income (or output), which satisfies the consumption Euler condition

$$y_t^f = \mathrm{E}_t y_{t+1}^f - \frac{1}{\hat{\sigma}}\left[i_t^f - \mathrm{E}_t \pi_{t+1}^f\right], \tag{A19}$$

as $y_t^f = c_t^f$.

A.4 Equilibrium

The resource constraint requires that the production of domestic goods satisfies

$$y_t \;=\; (1-\omega)c_t + (2-\omega)\omega\delta e_t + \omega y_t^f, \tag{A20}$$

using (A14) and (A18). Alternatively, we can solve for c_t as

$$c_t = \frac{1}{1-\omega} y_t - \frac{(2-\omega)\omega\delta}{1-\omega} e_t - \frac{\omega}{1-\omega} y_t^f. \tag{A21}$$

Combining with the consumption Euler equation (A3) we obtain

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1-\omega}{\hat{\sigma}} \left[i_t - \mathrm{E}_t \pi_{t+1}^c \right] - (2-\omega)\omega\delta \mathrm{E}_t \Delta e_{t+1} - \omega \mathrm{E}_t \Delta y_{t+1}^f. \tag{A22}$$

Combining the consumption Euler equation (A3) and the CPI inflation rate in equation (A11) implies that

$$i_t - \mathrm{E}_t \pi_{t+1} = \hat{\sigma} \mathrm{E}_t \Delta c_{t+1} + \omega \mathrm{E}_t \Delta e_{t+1}, \tag{A23}$$

and the foreign Euler equation implies that

$$i_t^f - \mathrm{E}_t \pi_{t+1}^f = \hat{\sigma} \mathrm{E}_t \Delta y_{t+1}^f. \tag{A24}$$

Using this in the UIP condition (A9), we obtain

$$\begin{aligned} e_t &= \mathrm{E}_t e_{t+1} - \hat{\sigma} \left[\mathrm{E}_t \Delta c_{t+1} - \mathrm{E}_t \Delta y_{t+1}^f \right] - \omega \mathrm{E}_t \Delta e_{t+1} + \phi_t \\ &= \mathrm{E}_t e_{t+1} - \frac{\hat{\sigma}}{1-\omega} \left[\mathrm{E}_t \Delta c_{t+1} - \mathrm{E}_t \Delta y_{t+1}^f \right] + \frac{1}{1-\omega} \phi_t, \end{aligned} \tag{A25}$$

which can be solved forward to obtain[21]

$$e_t = \frac{\hat{\sigma}}{1-\omega} \left[c_t - y_t^f \right] + \frac{1}{1-\omega} \phi_t. \tag{A26}$$

Using the expression for consumption in (A21) the real exchange rate is given by

$$\begin{aligned} e_t &= \frac{\hat{\sigma}}{1-\omega} \left[\frac{1}{1-\omega} y_t - \frac{(2-\omega)\omega\delta}{1-\omega} e_t - \frac{1}{1-\omega} y_t^f \right] + \frac{1}{1-\omega} \phi_t \\ &= \frac{\hat{\sigma}}{\Omega} \left[y_t - y_t^f \right] + \frac{1-\omega}{\Omega} \phi_t, \end{aligned} \tag{A27}$$

where

$$\Omega \equiv (1-\omega)^2 + (2-\omega)\omega\delta\hat{\sigma} > 0. \tag{A28}$$

Using this in (A21) gives

$$\begin{aligned} c_t &= \frac{1}{1-\omega} y_t - \frac{(2-\omega)\omega\delta}{1-\omega} \left[\frac{\hat{\sigma}}{\Omega} \left(y_t - y_t^f \right) + \frac{1-\omega}{\Omega} \phi_t \right] - \frac{\omega}{1-\omega} y_t^f \\ &= \frac{1-\omega}{\Omega} y_t + \frac{\Omega - (1-\omega)}{\Omega} y_t^f - \frac{(2-\omega)\omega\delta}{\Omega} \phi_t. \end{aligned} \tag{A29}$$

[21] We assume that $\lim_{j\to\infty} \mathrm{E}_t c_{t+j} = \lim_{j\to\infty} \mathrm{E}_t y_{t+j}^f = 0$, and that for all j, $\mathrm{E}_t \phi_{t+j} = 0$.

Using equation (A12) and the log-linearized version of the production function in equation (A15), $y_t = a_t + n_t$, we can write then marginal cost in equation (A17) as

$$v_t = v_y y_t + v_a a_t + v_\phi \phi_t + v_{yf} y_t^f, \tag{A30}$$

where

$$\begin{aligned} v_y &\equiv \frac{\eta\Omega + \hat{\sigma}}{\Omega} > 0, && \text{(A31)} \\ v_a &\equiv -(1+\eta) < 0, && \text{(A32)} \\ v_\phi &\equiv \frac{1-\omega-\Omega}{\Omega} \\ &= \frac{\omega(1-\omega) - (2-\omega)\omega\delta\hat{\sigma}}{\Omega}, && \text{(A33)} \\ v_{yf} &\equiv \frac{\hat{\sigma}(\Omega-1)}{\Omega}. && \text{(A34)} \end{aligned}$$

Note that v_ϕ is negative if

$$\delta\hat{\sigma} > \frac{1-\omega}{2-\omega}, \tag{A35}$$

and a sufficient (but not necessary) condition for this inequality to be satisfied is that $\delta\hat{\sigma} > 1/2$, which is typically the case.

A.5 Flexible-price equilibrium

Denoting by $\bar{z}$ the flexible-price level of the variable z, the flexible-price equilibrium is characterized by $\bar{v}_t = \bar{\phi}_t = 0$. Thus, equations (A30) and (A27) imply that the flexible-price levels of output and the real exchange rate are

$$\begin{aligned} \bar{y}_t &= -\frac{v_a}{v_y} a_t - \frac{v_{yf}}{v_y} y_t^f, && \text{(A36)} \\ \bar{e}_t &= \frac{\hat{\sigma}}{\Omega}\left[\bar{y}_t - y_t^f\right], && \text{(A37)} \end{aligned}$$

while the deviation of the real exchange rate from its flexible-price level is given by

$$e_t - \bar{e}_t = \frac{\hat{\sigma}}{\Omega}\left[y_t - \bar{y}_t\right] + \frac{1-\omega}{\Omega}\phi_t. \tag{A38}$$

A.6 The final steps

We can then write marginal cost in equation (A30) as

$$v_t = \hat{v}_y x_t + \hat{v}_e \left[e_t - \bar{e}_t\right], \tag{A39}$$

where $x_t \equiv y_t - \bar{y}_t$ is the output gap, and

$$\begin{aligned} \hat{v}_y &= v_y - \frac{\hat{\sigma}}{1-\omega} v_\phi \\ &= \frac{(1-\omega)\eta + \hat{\sigma}}{1-\omega} > 0, \end{aligned} \tag{A40}$$

$$\begin{aligned} \hat{v}_e &= \frac{\Omega}{1-\omega} v_\phi \\ &= \frac{1-\omega-\Omega}{1-\omega} \\ &= \frac{\omega(1-\omega) - (2-\omega)\omega\delta\hat{\sigma}}{1-\omega}, \end{aligned} \tag{A41}$$

where $\hat{v}_e$ is typically negative.

This implies that the Phillips curve (A16) can be expressed as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \hat{\kappa} \hat{v}_y x_t + \hat{\kappa} \hat{v}_e \left[e_t - \bar{e}_t \right], \tag{A42}$$

and the Euler equation (A22) can be written as

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - \frac{1-\omega}{\hat{\sigma}} \left[i_t - \mathrm{E}_t \pi^c_{t+1} \right] - (2-\omega)\omega\delta \mathrm{E}_t \Delta e_{t+1} \\ &\quad + \mathrm{E}_t \Delta \bar{y}_{t+1} - \omega \mathrm{E}_t \Delta y^f_{t+1} \\ &= \mathrm{E}_t x_{t+1} - \frac{1-\omega}{\hat{\sigma}} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] - \frac{\Omega - (1-\omega)}{\hat{\sigma}} \mathrm{E}_t \Delta e_{t+1} \\ &\quad + \mathrm{E}_t \Delta \bar{y}_{t+1} - \omega \mathrm{E}_t \Delta y^f_{t+1}. \end{aligned} \tag{A43}$$

Finally, setting all foreign variables to zero, equations (A42), (A43), and (A9) give a complete description of the small open economy:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t - \alpha e_t + \Sigma_\pi \varepsilon^\pi_t, \tag{A44}$$

$$x_t = \mathrm{E}_t x_{t+1} - \frac{1}{\sigma} \left[i_t - \mathrm{E}_t \pi_{t+1} \right] - \gamma \left[\mathrm{E}_t e_{t+1} - e_t \right] + \Sigma_x \varepsilon^x_t, \tag{A45}$$

$$e_t = \mathrm{E}_t e_{t+1} - \left[i_t - \mathrm{E}_t \pi_{t+1} \right] + \Sigma_e \varepsilon^e_t, \tag{A46}$$

where

$$\begin{aligned} \kappa &\equiv \hat{\kappa} \hat{v}_y \\ &= \hat{\kappa} \frac{(1-\omega)\eta + \hat{\sigma}}{1-\omega} > 0, \end{aligned} \tag{A47}$$

$$\begin{aligned} \alpha &\equiv -\hat{\kappa} \hat{v}_e \\ &= \hat{\kappa} \frac{\Omega - (1-\omega)}{1-\omega}, \end{aligned} \tag{A48}$$

$$\sigma \equiv \frac{\hat{\sigma}}{1-\omega} > 0, \tag{A49}$$

$$\gamma \equiv \frac{\Omega - (1-\omega)}{\hat{\sigma}}, \tag{A50}$$

$$\varepsilon_t^\pi \equiv -\frac{\alpha}{\Sigma_\pi}\bar{e}_t, \tag{A51}$$

$$\varepsilon_t^x \equiv \frac{1}{\Sigma_x}\mathrm{E}_t\left[\bar{y}_{t+1} - \bar{y}_t\right], \tag{A52}$$

$$\varepsilon_t^e \equiv \frac{1}{\Sigma_e}\phi_t. \tag{A53}$$

Note that α and γ are both positive as long as $\Omega - (1 - \omega) > 0$, which we showed above is typically satisfied.

B The reduced form

First, it is useful to define

$$\Theta_j^{-1} \equiv \frac{\Sigma_j}{(\gamma + \sigma^{-1})\theta_j} > 0, \tag{B1}$$

for $j = \pi, x, e$, and we note that $\lim_{\theta_j \to \infty} \Theta_j^{-1} = 0$.

B.1 The worst-case model

To find the reduced from for inflation and the exchange rate in the worst-case model, first write equations (30) and (32) as

$$\begin{aligned} \pi_t &= a_1 \mathrm{E}_t \pi_{t+1} + a_2 e_t + a_3 \varepsilon_t^{\pi}, & \text{(B2)} \\ e_t &= \mathrm{E}_t e_{t+1} + c_1 \mathrm{E}_t \pi_{t+1} + c_2 \pi_t + c_3 \varepsilon_t^{x} + c_4 \varepsilon_t^{e}, & \text{(B3)} \end{aligned}$$

where

$$\begin{aligned} a_1 &\equiv \frac{\beta}{C}, \quad a_2 \equiv -\frac{\alpha}{C}, \quad a_3 \equiv \frac{\Sigma_\pi}{C}, & \text{(B4)} \\ c_1 &\equiv \frac{\sigma A}{1+\sigma\gamma}, \quad c_2 \equiv -\frac{D}{1+\sigma\gamma}, \quad c_3 \equiv -\frac{\sigma \Sigma_x}{1+\sigma\gamma}, \quad c_4 \equiv \frac{\Sigma_e}{1+\sigma\gamma}, & \text{(B5)} \end{aligned}$$

and we seek a solution of the form

$$\begin{aligned} \pi_t &= a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e}, & \text{(B6)} \\ e_t &= c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e}, & \text{(B7)} \end{aligned}$$

where the a_j, c_j coefficients remain to be determined.

Setting expectations to zero and combining (B2)–(B3) with (B6)–(B7) we obtain

$$\begin{aligned} a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e} &= a_2 \left[c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e} \right] + a_3 \varepsilon_t^{\pi}, & \text{(B8)} \\ c_\pi \varepsilon_t^{\pi} + c_x \varepsilon_t^{x} + c_e \varepsilon_t^{e} &= c_2 \left[a_\pi \varepsilon_t^{\pi} + a_x \varepsilon_t^{x} + a_e \varepsilon_t^{e} \right] + c_3 \varepsilon_t^{x} + c_4 \varepsilon_t^{e}. & \text{(B9)} \end{aligned}$$

Thus, the coefficients satisfy

$$\begin{aligned} a_\pi &= a_2 c_\pi + a_3, & \text{(B10)} \\ a_x &= a_2 c_x, & \text{(B11)} \\ a_e &= a_2 c_e, & \text{(B12)} \\ c_\pi &= c_2 a_\pi, & \text{(B13)} \\ c_x &= c_2 a_x + c_3, & \text{(B14)} \\ c_e &= c_2 a_e + c_4, & \text{(B15)} \end{aligned}$$

and the solution of this system is

$$
\begin{aligned}
a_\pi &= a_2 c_2 a_\pi + a_3 \\
&= \frac{a_3}{1 - a_2 c_2}, && \text{(B16)} \\
c_\pi &= \frac{a_3 c_2}{1 - a_2 c_2}, && \text{(B17)} \\
c_x &= c_2 a_2 c_x + c_3 \\
&= \frac{c_3}{1 - a_2 c_2}, && \text{(B18)} \\
a_x &= \frac{a_2 c_3}{1 - a_2 c_2}, && \text{(B19)} \\
c_e &= c_2 a_2 c_e + c_4 \\
&= \frac{c_4}{1 - a_2 c_2}, && \text{(B20)} \\
a_e &= \frac{a_2 c_4}{1 - a_2 c_2}. && \text{(B21)}
\end{aligned}
$$

The reduced-form coefficients are then given by

$$
\begin{aligned}
a_\pi &= \frac{a_3}{1 - a_2 c_2} = \frac{(1 + \sigma\gamma)\Sigma_\pi}{E}, && \text{(B22)} \\
a_x &= \frac{a_2 c_3}{1 - a_2 c_2} = \frac{\alpha\sigma\Sigma_x}{E}, && \text{(B23)} \\
a_e &= \frac{a_2 c_4}{1 - a_2 c_2} = -\frac{\alpha\Sigma_e}{E}, && \text{(B24)} \\
c_\pi &= \frac{a_3 c_2}{1 - a_2 c_2} = -\frac{D\Sigma_\pi}{E}, && \text{(B25)} \\
c_x &= \frac{c_3}{1 - a_2 c_2} = -\frac{\sigma C\Sigma_x}{E}, && \text{(B26)} \\
c_e &= \frac{c_4}{1 - a_2 c_2} = \frac{C\Sigma_e}{E}, && \text{(B27)}
\end{aligned}
$$

where

$$
\begin{aligned}
E &\equiv (1 - a_1 c_1)(1 + \sigma\gamma)C \\
&= (1 + \sigma\gamma)C - \alpha D \\
&= (1 + \sigma\gamma)\left[1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi}\right] - \alpha\left[\sigma A + \alpha\sigma\Sigma_x\Theta_x^{-1} + \alpha\sigma^{-1}\Sigma_e\Theta_e^{-1}\right] \\
&= 1 + \sigma\gamma + [\kappa + \sigma(\gamma\kappa - \alpha)]\,A - (1 + \sigma\gamma)\frac{\Sigma_\pi^2}{\theta_\pi} \\
&\quad -\alpha^2\sigma\Sigma_x\Theta_x^{-1} - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1} > 0, && \text{(B28)}
\end{aligned}
$$

which is positive, as

$$\kappa + \sigma(\gamma\kappa - \alpha) = \frac{\hat{\kappa}}{1-\omega}\left[\hat{\sigma} + \eta\Omega\right] > 0. \tag{B29}$$

B.2 The policy rule

Using the interest rate equation (27), the reduced form for the interest rate is

$$\begin{aligned} i_t &= \sigma B \pi_t + \sigma\gamma e_t + \sigma\Sigma_x \varepsilon_t^x, \\ &= d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e, \end{aligned} \tag{B30}$$

where

$$\begin{aligned} d_\pi &= \sigma\left[B a_\pi + \gamma c_\pi\right] \\ &= F_\pi a_\pi > 0, && \text{(B31)} \\ d_x &= \sigma\left[B a_x + \gamma c_x + \Sigma_x\right] \\ &= F_x a_\pi + \sigma\Sigma_x > 0, && \text{(B32)} \\ d_e &= \sigma\left[B a_e + \gamma c_e\right] \\ &= F_e a_\pi > 0, && \text{(B33)} \end{aligned}$$

where

$$\begin{aligned} F_\pi &\equiv \frac{\sigma}{1+\sigma\gamma} B - \frac{\alpha\gamma}{1+\sigma\gamma}\Sigma_e\Theta_e^{-1} > 0, && \text{(B34)} \\ F_x &\equiv \left[\alpha B - \gamma C\right]\frac{\sigma^2}{1+\sigma\gamma}\frac{\Sigma_x}{\Sigma_\pi} < 0, && \text{(B35)} \\ F_e &\equiv \left[\gamma C - \alpha B\right]\frac{\sigma}{1+\sigma\gamma}\frac{\Sigma_e}{\Sigma_\pi} > 0, && \text{(B36)} \end{aligned}$$

where we used

$$\begin{aligned} \alpha B - \gamma C &= \alpha A + \alpha^2\Sigma_x\Theta_x^{-1} - \gamma - \kappa\gamma A - \gamma\frac{\Sigma_\pi^2}{\theta_\pi} \\ &= -\gamma + (\alpha - \kappa\gamma)A + \alpha^2\Sigma_x\Theta_x^{-1} - \gamma\frac{\Sigma_\pi^2}{\theta_\pi} \\ &= -\gamma - \hat{\kappa}\eta\gamma A - \gamma\frac{\Sigma_\pi^2}{\theta_\pi} + \alpha^2\Sigma_x\Theta_x^{-1} < 0, && \text{(B37)} \end{aligned}$$

and

$$\begin{aligned} \alpha - \kappa\gamma &= \frac{\hat{\kappa}\hat{\sigma}\gamma}{1-\omega} - \gamma\hat{\kappa}\frac{\eta(1-\omega)+\hat{\sigma}}{1-\omega} \\ &= \frac{\hat{\kappa}\gamma}{1-\omega}\left[\hat{\sigma} - \eta(1-\omega) - \hat{\sigma}\right] \end{aligned}$$

$$= -\hat{\kappa}\gamma\eta. \tag{B38}$$

To see that $d_x > 0$, note that

$$\begin{aligned} F_x a_\pi + \sigma\Sigma_x &= [\alpha B - \gamma C] \frac{\sigma^2}{1+\sigma\gamma} \frac{\Sigma_x}{\Sigma_\pi} \frac{(1+\sigma\gamma)\Sigma_\pi}{E} + \sigma\Sigma_x \\ &= [\alpha\sigma B - \sigma\gamma C + E] \frac{\sigma\Sigma_x}{E} > 0. \end{aligned} \tag{B39}$$

B.3 The approximating model

It is useful to note that

$$\begin{aligned} & E + \alpha\sigma B - \sigma\gamma C \\ &= (1+\sigma\gamma)C - \alpha D + \alpha\sigma B - \sigma\gamma C \\ &= C + \alpha(\sigma B - D) \\ &= C - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1} > 0, \end{aligned} \tag{B40}$$

To find the solution for the approximating model, use the policy rule (70) in the equations for inflation, output and the exchange rate, setting the misspecification to zero:

$$i_t = d_\pi \varepsilon_t^\pi + d_x \varepsilon_t^x + d_e \varepsilon_t^e, \tag{B41}$$

$$\pi_t = \kappa x_t - \alpha e_t + \Sigma_\pi \varepsilon_t^\pi, \tag{B42}$$

$$x_t = -\sigma^{-1} i_t + \gamma e_t + \Sigma_x \varepsilon_t^x, \tag{B43}$$

$$e_t = -i_t + \Sigma_e \varepsilon_t^e. \tag{B44}$$

The solution is

$$\pi_t = \bar{a}_\pi \varepsilon_t^\pi + \bar{a}_x \varepsilon_t^x + \bar{a}_e \varepsilon_t^e, \tag{B45}$$

$$x_t = \bar{b}_\pi \varepsilon_t^\pi + \bar{b}_x \varepsilon_t^x + \bar{b}_e \varepsilon_t^e, \tag{B46}$$

$$e_t = \bar{c}_\pi \varepsilon_t^\pi + \bar{c}_x \varepsilon_t^x + \bar{c}_e \varepsilon_t^e, \tag{B47}$$

where

$$\bar{c}_\pi = -d_\pi < 0, \tag{B48}$$

$$\bar{c}_x = -d_x < 0, \tag{B49}$$

$$\begin{aligned} \bar{c}_e &= \Sigma_e - d_e \\ &= [E + \alpha\sigma B - \sigma\gamma C] \frac{\Sigma_e}{E} > 0, \end{aligned} \tag{B50}$$

$$\bar{b}_\pi = \gamma \bar{c}_\pi - \sigma^{-1} d_\pi$$

$$
\begin{aligned}
&= -(\gamma+\sigma^{-1})d_\pi < 0, && \text{(B51)}\\
\bar{b}_x &= \Sigma_x + \gamma\bar{c}_x - \sigma^{-1}d_x \\
&= \Sigma_x - (\gamma+\sigma^{-1})d_x, \\
&= \{E - (1+\sigma\gamma)\left[E + \alpha\sigma B - \sigma\gamma C\right]\}\frac{\Sigma_x}{E} \\
&= \left[-\alpha D + (\gamma+\sigma^{-1})\alpha^2\Sigma_e\Theta_e^{-1}\right]\frac{\Sigma_x}{E} < 0, && \text{(B52)}\\
\bar{b}_e &= \gamma\bar{c}_e - \sigma^{-1}d_e \\
&= \gamma\Sigma_e - (\gamma+\sigma^{-1})d_e, \\
&= \{\gamma E + (1+\sigma\gamma)\left[\alpha B - \gamma C\right]\}\frac{\Sigma_e}{E} \\
&= \{\gamma\left[E + \alpha\sigma B - \sigma\gamma C\right] + \left[\alpha B - \gamma C\right]\}\frac{\Sigma_e}{E} \\
&= \left[\alpha B - \gamma\alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1}\right]\frac{\Sigma_e}{E} > 0, && \text{(B53)}\\
\bar{a}_\pi &= \Sigma_\pi + \kappa\bar{b}_\pi - \alpha\bar{c}_\pi \\
&= \Sigma_\pi + \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]d_\pi \\
&= \left\{E + \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]\left[\sigma B - \alpha\gamma\Sigma_e\Theta_e^{-1}\right]\right\}\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)C - \alpha\left[\sigma B + \alpha\sigma^{-1}\Sigma_e\Theta_e^{-1}\right] \\
&\quad + \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]\left[\sigma B - \alpha\gamma\Sigma_e\Theta_e^{-1}\right]\Bigg\}\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)\left[1 + \kappa A - \frac{\Sigma_\pi^2}{\theta_\pi}\right] - \kappa(1+\sigma\gamma)\left[A + \alpha\Sigma_x\Theta_x^{-1}\right] \\
&\quad - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1} - \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]\alpha\gamma\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_\pi}{E} \\
&= \Bigg\{(1+\sigma\gamma)\left[1 - \frac{\Sigma_\pi^2}{\theta_\pi}\right] - \kappa(1+\sigma\gamma)\alpha\Sigma_x\Theta_x^{-1} - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1} \\
&\quad - \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]\alpha\gamma\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_\pi}{E} > 0, && \text{(B54)}\\
\bar{a}_x &= \kappa\bar{b}_x - \alpha\bar{c}_x \\
&= \kappa\Sigma_x + \left[\alpha - \kappa(\gamma+\sigma^{-1})\right]d_x \\
&= \left\{\kappa E + \left[\alpha\sigma - \kappa(1+\sigma\gamma)\right]\left[C - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1}\right]\right\}\frac{\Sigma_x}{E} \\
&= \left\{\kappa\left[(1+\sigma\gamma)C - \alpha D\right] + \left[\alpha\sigma - \kappa(1+\sigma\gamma)\right]\left[C - \alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1}\right]\right\}\frac{\Sigma_x}{E} \\
&= \Bigg\{\alpha\left[\sigma + \sigma\kappa A - \sigma\frac{\Sigma_\pi^2}{\theta_\pi} - \sigma\kappa A - \alpha\kappa\sigma\Sigma_x\Theta_x^{-1} - \alpha\kappa\sigma^{-1}\Sigma_e\Theta_e^{-1}\right] \\
&\quad - \left[\alpha\sigma - \kappa(1+\sigma\gamma)\right]\alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1}\Bigg\}\frac{\Sigma_x}{E}
\end{aligned}
$$

$$= \left\{\alpha\sigma\left[1-\frac{\Sigma_\pi^2}{\theta_\pi}-\alpha\kappa\Sigma_x\Theta_x^{-1}\right]-[\alpha-\kappa\gamma]\,\alpha^2\Sigma_e\Theta_e^{-1}\right\}\frac{\Sigma_x}{E}>0, \tag{B55}$$

$$\begin{aligned}
\bar{a}_e &= \kappa\bar{b}_e-\alpha\bar{c}_e \\
&= (\kappa\gamma-\alpha)\Sigma_e+\left[\alpha-\kappa(\gamma+\sigma^{-1})\right]d_e \\
&= \left\{(\kappa\gamma-\alpha)E+\sigma\left[\alpha-\kappa(\gamma+\sigma^{-1})\right][\gamma C-\alpha B]\right\}\frac{\Sigma_e}{E} \\
&= \{(\kappa\gamma-\alpha)\,[E+\alpha\sigma B-\sigma\gamma C]+\kappa\,[\alpha B-\gamma C]\}\,\frac{\Sigma_e}{E} \\
&= \left\{(\kappa\gamma-\alpha)\left[C-\sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1}\right]+\kappa\,[\alpha B-\gamma C]\right\}\frac{\Sigma_e}{E} \\
&= \left\{\alpha\,[\kappa B-C]-(\kappa\gamma-\alpha)\sigma^{-1}\alpha^2\Sigma_e\Theta_e^{-1}\right\}\frac{\Sigma_e}{E}
\end{aligned}$$

$$= \left\{\alpha\left[\frac{\Sigma_\pi^2}{\theta_\pi}+\alpha\kappa\Sigma_x\Theta_x^{-1}-1\right]-(\kappa\gamma-\alpha)\alpha^2\sigma^{-1}\Sigma_e\Theta_e^{-1}\right\}\frac{\Sigma_e}{E}<0. \tag{B56}$$

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Figure 1: Effects of an increased preference for robustness on the coefficients in the worst-case model for inflation

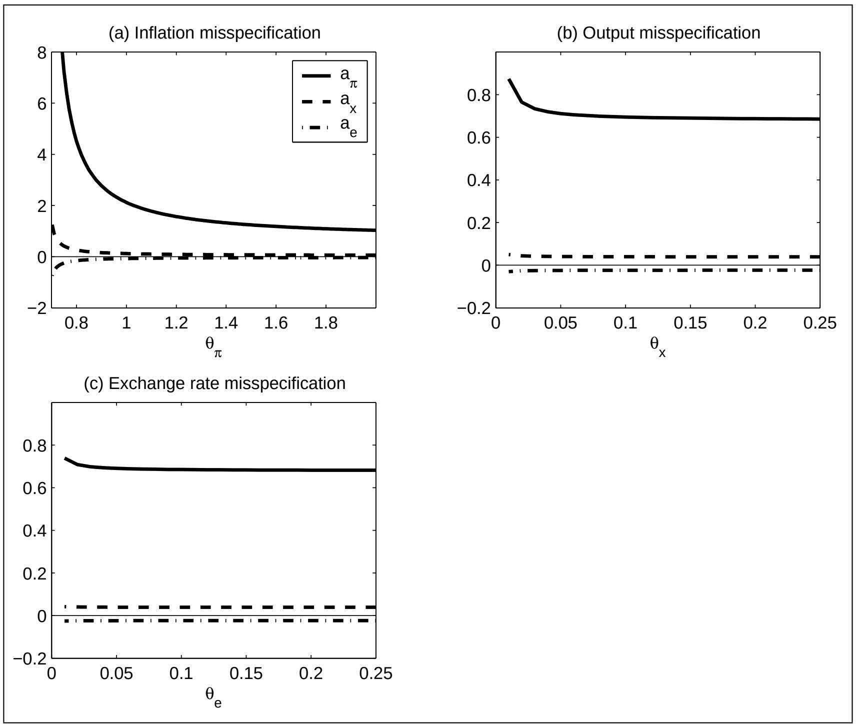

Figure 2: Effects of an increased preference for robustness on the coefficients in the worst-case model for output

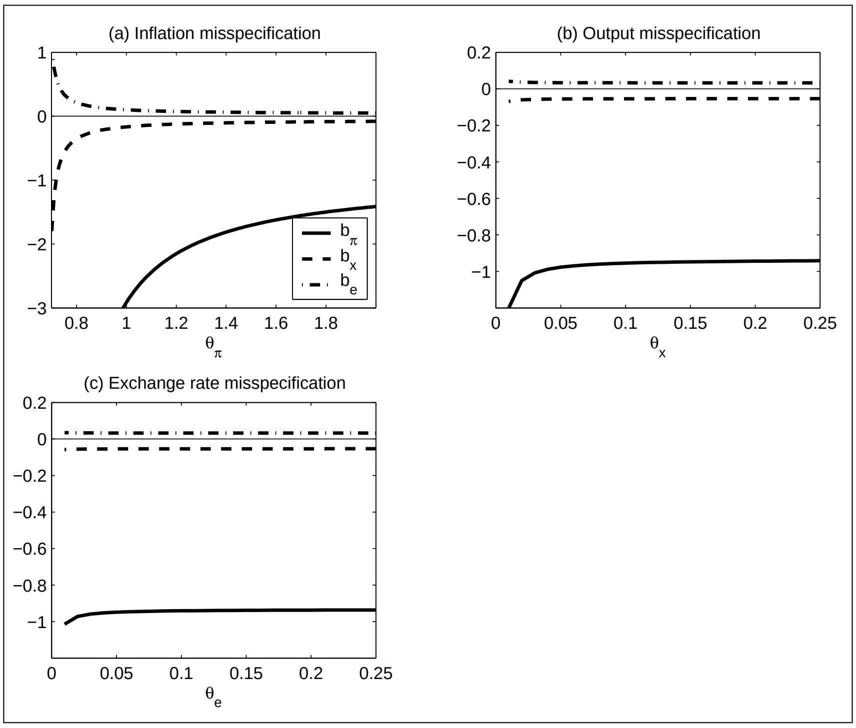

Figure 3: Effects of an increased preference for robustness on the coefficients in the worst-case model for the exchange rate

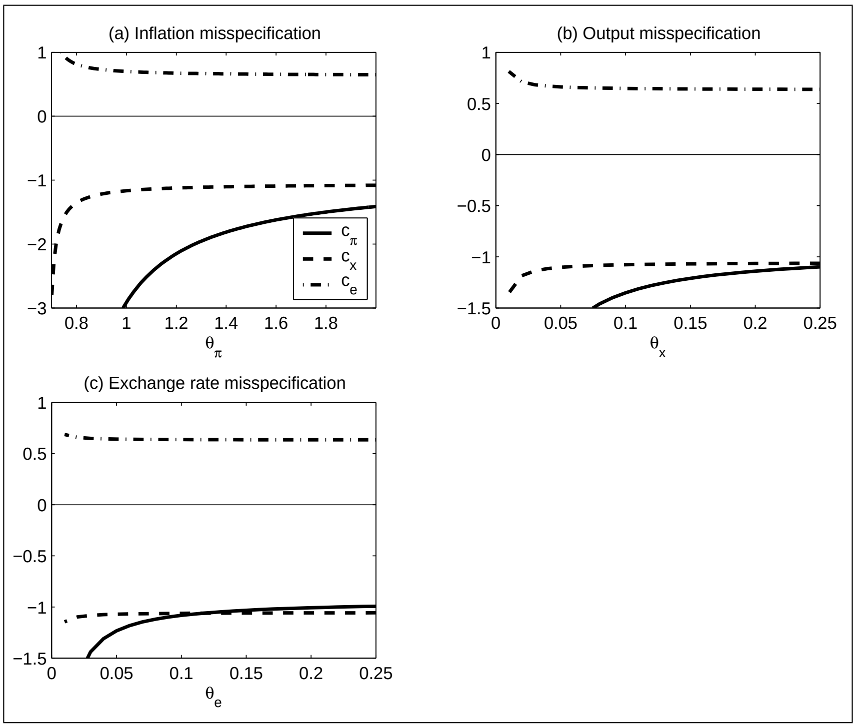

Figure 4: Effects of an increased preference for robustness on the coefficients in the policy rule

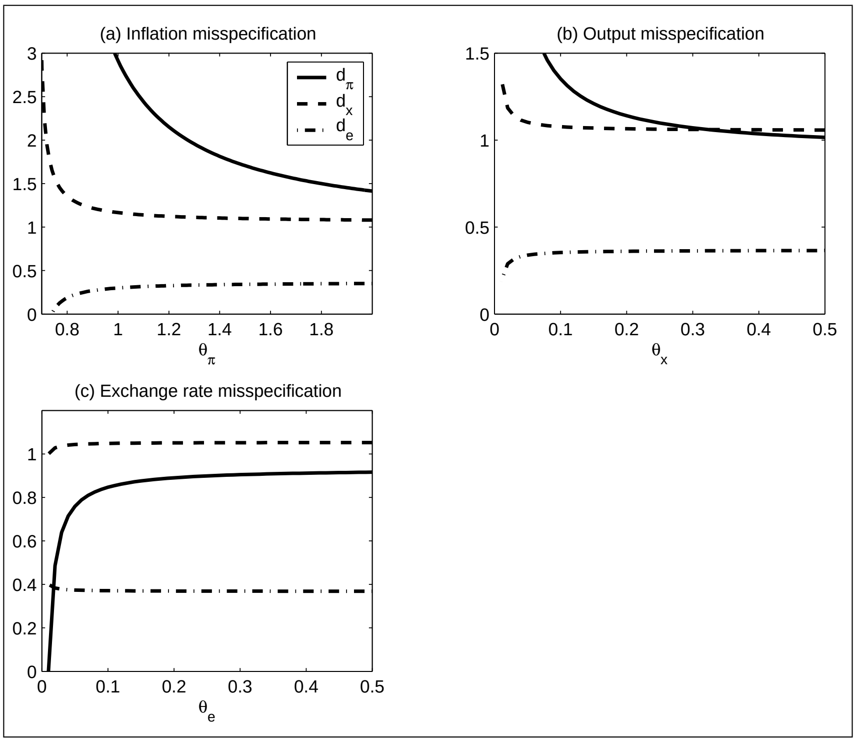

Figure 5: Effects of an increased preference for robustness on the coefficients in the approximating model for inflation

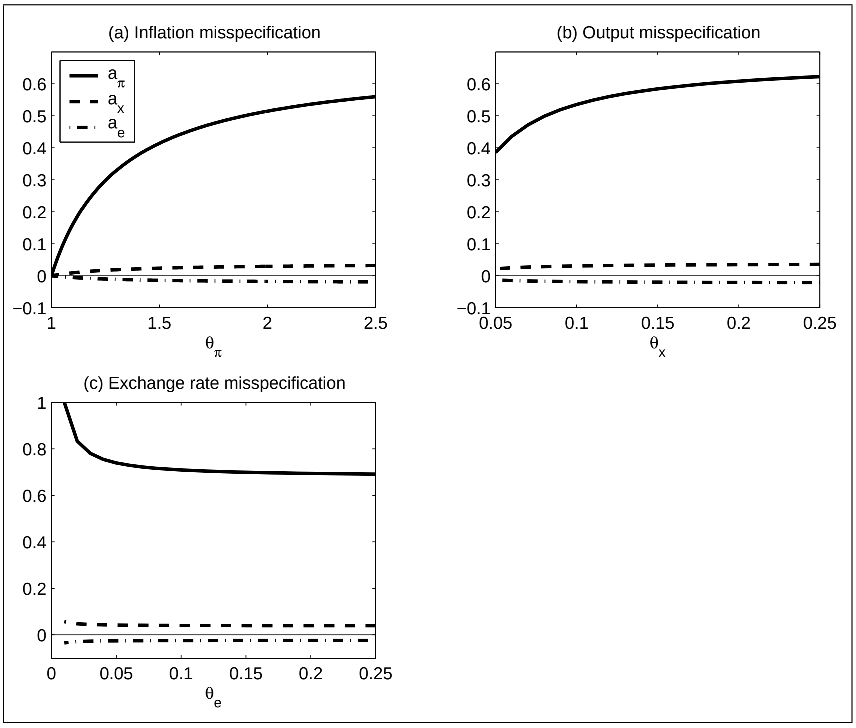

Figure 6: Effects of an increased preference for robustness on the coefficients in the approximating model for output

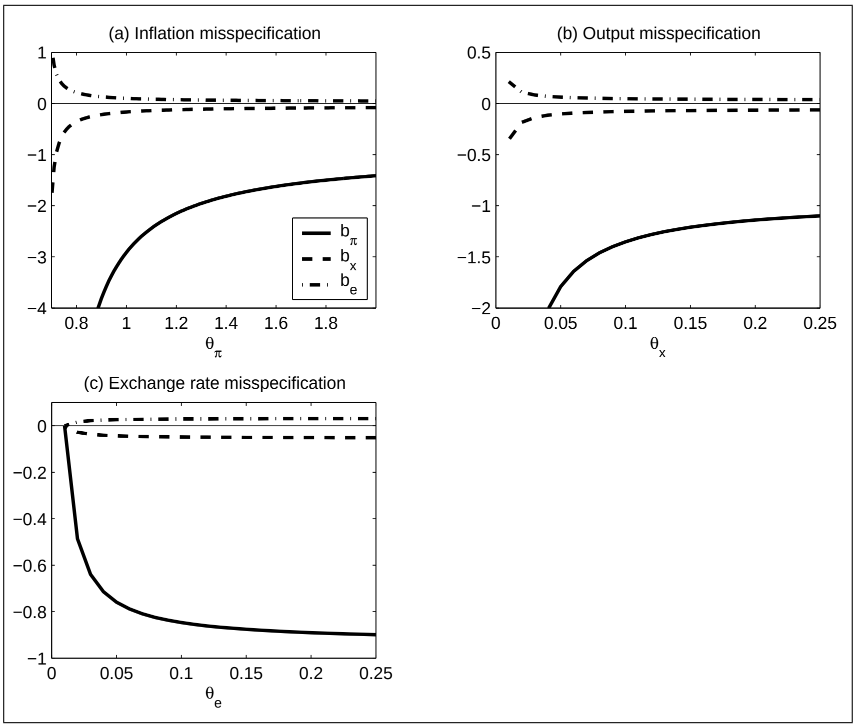

Figure 7: Effects of an increased preference for robustness on the coefficients in the approximating model for the exchange rate

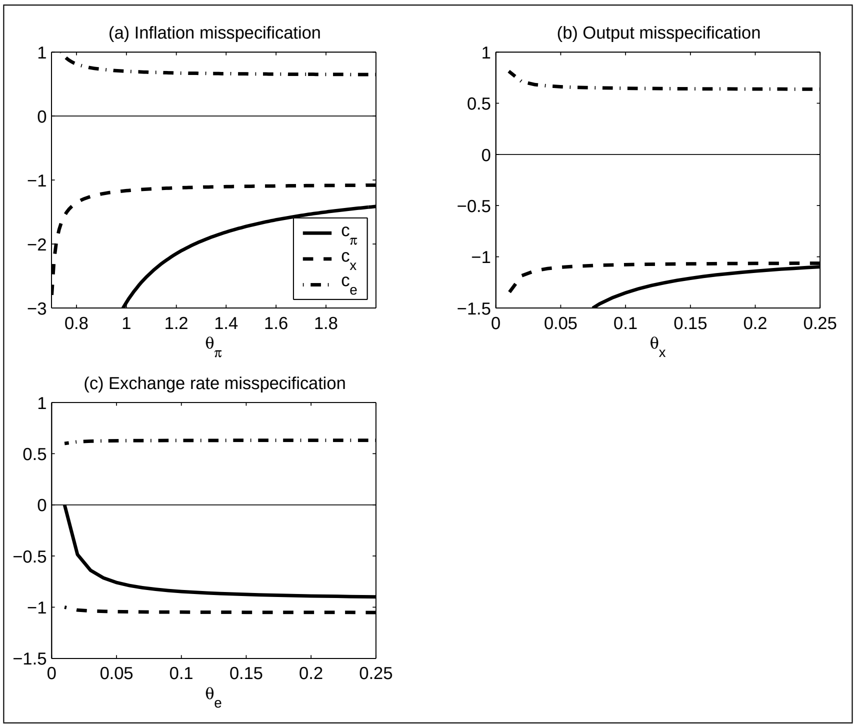

Figure 8: Effects of an increased preference for robustness on variances in the worst-case model

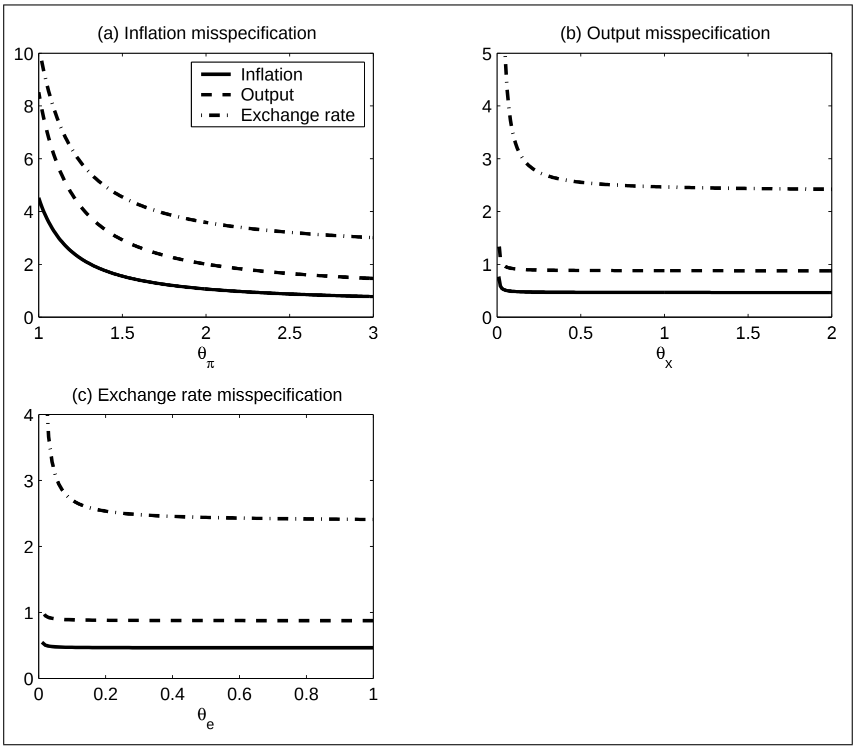

Figure 9: Effects of an increased preference for robustness on the variance of the interest rate

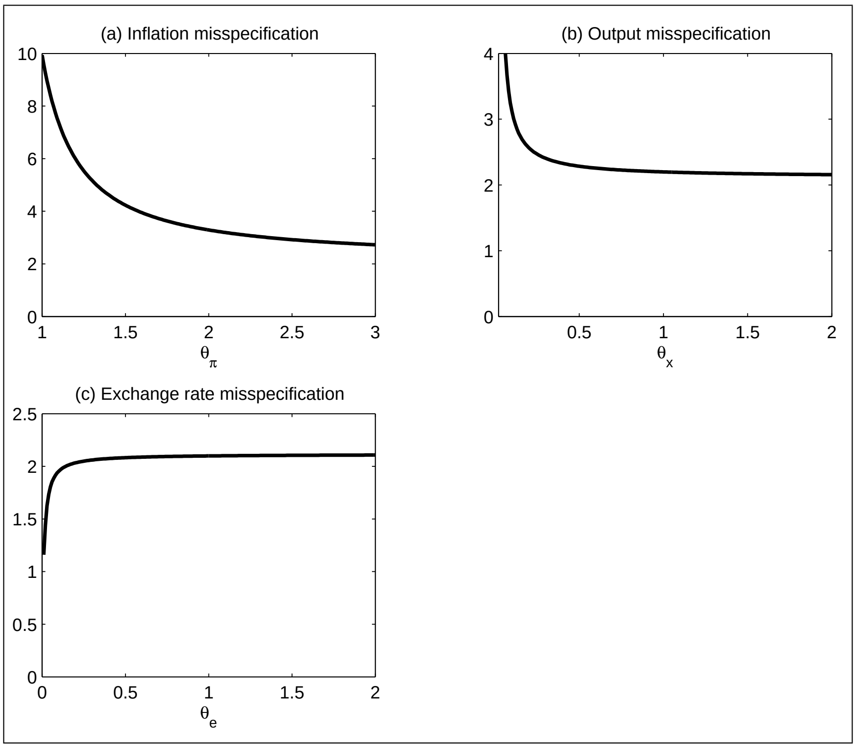

Figure 10: Effects of an increased preference for robustness on variances in the approximating model