Monetary Policy in a Small Open Economy with a Preference for Robustness

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Abstract

We use robust control techniques to study the effects of model uncertainty on monetary policy in a small-open-economy model estimated on Australian data. Compared to the closed economy, the presence of open-economy transmission channels and shocks not only produces new trade-offs for monetary policy, but also introduces additional sources of specification errors. We find that price markup shocks in the domestic and import sector are important contributors to volatility in the model, and that the domestic and import sector Phillips curves are particularly vulnerable to model misspecification. On the other hand, deviations from the interest rate parity condition do not contribute much to overall volatility, nor is the parity condition especially vulnerable to misspecification. Our results suggest that it may be more important for central banks in small open economies to understand the nature of price setting and the effects of exchange rate movements on the economy than the determination of the exchange rate itself.

Keywords: Model uncertainty, Model misspecification, Robust control.

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1 Introduction

Although the canonical New Keynesian model (Goodfriend and King (1997), Clarida, Galí, and Gertler (1999), and Woodford (2003)) is used extensively to analyze monetary policy, important questions about its structure remain unresolved. There are ongoing debates about the role of forward-looking inflation expectations, about the nature of the driving variable—real marginal cost or an output gap—in the New Keynesian Phillips curve, and about the importance of habit formation and consumption smoothing in the forward-looking "IS" curve. More generally, it is widely perceived among practitioners that monetary policy affects the economy with "long and variable lags" in ways that models generally do not acknowledge.

Of course, these debates about the appropriate structure of closed-economy New Keynesian models apply equally to open-economy specifications. After all, the transmission mechanisms that operate in open-economy models are often similar to those present in closed-economy specifications. However, unlike in the closed economy, in the open economy there can be concerns about the level of exchange rate pass-through, concerns centered around whether pass-through is full or partial, and about the extent to which imports are consumed or employed as intermediate inputs in the production of domestic goods. Similarly, exchange rate dynamics are difficult to model and from an empirical standpoint there is good reason to view uncovered interest rate parity with suspicion. Importantly, these concerns extend beyond parameter uncertainty, amounting to a concern about the very structure of the model used to describe the economy.

We study the conduct of monetary policy in a model of a small open economy developed by Justiniano and Preston (2008) and estimated on Australian data. Unlike most papers that consider the design of monetary policy in open-economy contexts, we introduce a concern for model misspecification on the part of the central bank and focus on policy rules that have been formulated purposefully to be robust to model misspecification. In the tradition of Hansen and Sargent (2008), we assume that the central bank possesses a benchmark model of the economy, which it is concerned may be misspecified, but that it is unwilling to posit a probability distribution over possible specification errors. The central bank allows for specification errors that lie within a neighborhood of its benchmark specification and conducts monetary policy to guard against the worst-case specification error. In taking this approach, the central bank recognizes that its policy will be suboptimal if its benchmark model is actually specified correctly, but it still conducts policy this way, gaining comfort from the knowledge that by doing so it is insuring against catastrophic outcomes.

The open-economy model that we consider is based on the theoretical model of Monacelli (2005). The model allows households to consume goods produced both domestically and abroad, with sticky prices in both the domestic and the import sector. Sticky import prices imply that exchange rate movements do not feed directly through to consumer prices, that is, exchange rate pass-through is incomplete. The model also allows a portfolio allocation choice between domestic and foreign bonds, giving rise to an uncovered interest rate parity (UIP) condition and making the exchange rate an important channel for monetary policy and risk premium shocks an important source of economic volatility. As we show, the exchange rate channel introduces additional trade-offs that the central bank must acknowledge when formulating policy, and it introduces an additional location for possible model misspecification.

We contrast the sources of misspecification and the design of robust monetary policy with commitment by using several versions of our model: a closed-economy version, a version with open-economy transmission channels, but only domestic shocks, and versions with shocks emanating from the open-economy components of the model. We show that in a closed economy, a robust central bank should be concerned mainly with specification errors to the inflation equation (or Phillips curve). Adding open-economy transmission channels and shocks, we find that the relationship describing import price inflation is an important source of volatility and that it is also particularly prone to model misspecification. In contrast, shocks to the UIP condition are not a very important source of volatility, nor is the UIP condition particularly vulnerable to model misspecification. Thus, analogous to a closed economy, a central bank in a small open economy that is worried about model misspecification should be concerned mainly about the domestic and import sector Phillips curves. These results suggest that it may be more important for central banks in small open economies to understand the nature of price setting and the impact of exchange rate movements on import prices (that is, the degree of exchange rate pass-through) than the determination of the exchange rate itself and possible deviations from uncovered interest rate parity.

Our approach to robust monetary policy assumes that the central bank formulates policy to minimize the economic consequences of the worst-case specification errors. An alternative approach is for the central bank to build several models and to use these models to develop a policy that produces reasonable, if not optimal, outcomes in all of the models (Levin, Wieland, and Williams (1999, 2003)). Although this approach is intuitive and simple to implement, it is not necessarily the most attractive. The approach does not allow the central bank to address any concerns it may have about parameter uncertainty, it does not accommodate the possibility that agents other than the central bank may be concerned about model uncertainty, and it assumes that each of the models provides an equally plausible description of the economy. A second alternative is for the central bank to take a Bayesian approach, estimating a range of models and using Bayesian model averaging to evaluate competing policies (see Brock, Durlauf, and West (2007) and Batini, Justiniano, Levine, and Pearlman (2005)). The Bayesian approach does not assume that all of the models are equally plausible and it readily accommodates both parameter and model uncertainty, but it still does not easily allow all agents in the model to be concerned about model uncertainty. In contrast, the robust control approach has the advantages that the policymaker need only develop a single model and all agents in the economy can be concerned about model misspecification. Furthermore, the specification errors can reflect both model and parameter uncertainty.

Although model uncertainty—particularly uncertainty concerning exchange rate determination—is of obvious relevance for central banks in small open economies (see, for instance, West (2003)), surprisingly few studies have examined the issue. Leitemo and Söderström (2005) study the robustness of simple policy rules to uncertainty about exchange rate determination in a calibrated, stylized, small-open-economy model, concluding that a standard Taylor rule that responds to CPI inflation and the output gap performs well. They also argue that the Taylor rule is more robust to uncertainty about the formation of exchange rate expectations than are rules that respond to exchange rate movements. Batini, Justiniano, Levine, and Pearlman (2005) study the effects of Bayesian model uncertainty on monetary policy in an estimated two-country model. Unlike our study, they focus on large open economies and investigate the gains to policy coordination. Justiniano and Preston (2008) analyze the effects of parameter uncertainty on optimized Taylor-type rules for monetary policy in the model used here, but estimated on data not only from Australia, but also from Canada, and New Zealand. Using a Bayesian approach, they find that parameter uncertainty has small effects on the optimized monetary policy rules.

These papers all study specific types of model uncertainty without allowing private agents to have doubts about model specification. In contrast, we study more general forms of model uncertainty using robust control techniques that allow the central bank to formulate a policy that accommodates the effect model misspecification may have on private agents. Along similar lines, Lees (2006) analyzes a stylized small-open-economy model and finds that robust policies are generally more aggressive in response to shocks and that they imply less interest rate inertia. For his calibration and with discretionary policy, Lees (2006) concludes that the exchange rate is an important source of specification errors, and that the consequences of these specification errors outweigh the benefits to the central bank of exploiting the exchange rate channel to stabilize the economy. We instead study optimal policy with commitment within a completely microfounded model. We show that with policy set under commitment, misspecification in exchange rate determination is not very damaging. Finally, Leitemo and Söderström (2008b) present an analytic treatment of robust control in a minimalist small-open-economy model. They show that by guarding against specification errors in either the supply or demand side of the model the central bank raises the volatility of output and the exchange rate, whereas by guarding against specification errors in the exchange rate equation the central bank raises the volatility of inflation. We study a more general estimated model, with inertia in consumption and inflation, that is better suited to quantifying the effects of robustness in the small open economy.

The remainder of the paper is organized as follows. We first describe the model in Section 2. We then present our robust control algorithm in Section 3. We apply this algorithm to different versions of the model in Section 4, isolating the effects on robust policymaking of the open-economy policy channels and the open-economy shocks before studying the complete open-economy specification. Finally, we conclude in Section 5.

2 The model

Our model is based on the New Keynesian small-open-economy model developed by Justiniano and Preston (2008), who extend the theoretical model of Monacelli (2005). In this model households consume goods produced both domestically and abroad, with staggered price-setting in both the domestic and the import sector. With imported goods subject to price rigidity, and with importers pricing to market, the model can reproduce the incomplete exchange rate pass-through widely found to characterize the behavior of imported goods prices following exchange rate shocks (Campa and Goldberg (2005)). As there is ample evidence supporting incomplete exchange rate pass-through, allowing for sticky imported goods prices seems reasonable, especially since it is likely to be important for the design of monetary policy. A second key feature of the model is that it is not possible to achieve full price stability by setting the output gap to zero. The interest rate policy required to generate a zero output gap destabilizes inflation through its influence on imported goods prices.

The theoretical specification of Monacelli (2005) provides a simple microfounded description of private-sector behavior in an economy where goods prices are sticky. However, the model abstracts from the information and decision lags that can give rise to gradual adjustments and inertial responses to shocks. Justiniano and Preston (2008) therefore extend the model to allow for partial indexation of prices to inflation and habits in consumer preferences.

The model features five groups of agents: households, domestic-good firms, import firms, a central bank, and a foreign sector. We here only present the main characteristics of the model and the log-linearized equations. The reader is referred to Justiniano and Preston (2008) for more detail.

Households consume a basket containing both domestically produced and imported goods, save in nominal one-period bonds denoted in domestic or foreign currency, and supply labor to firms in the domestic sector. Household utility depends positively on consumption relative to an external habit stock and negatively on labor supply. When saving in foreign bonds, households pay an interest rate premium that depends on the domestic economy's net foreign asset position.

Denoting by c_t aggregate consumption, by r_t the interest rate on domestic nominal one-period bonds, and by $\pi_t \equiv p_t - p_{t-1}$ the rate of consumer price inflation (where p_t is the logarithm of the consumer price level), the household's intertemporal optimization problem leads to the consumption Euler equation

$$c_t - hc_{t-1} = \mathrm{E}_t c_{t+1} - hc_t - \frac{1-h}{\sigma}\left[r_t - \mathrm{E}_t\pi_{t+1} - u_t^g + \mathrm{E}_t u_{t+1}^g\right], \tag{1}$$

where u_t^g is a preference (or, equivalently, a discount factor) shock, h determines the importance of habits in consumption, and σ is the inverse of the elasticity of intertemporal substitution. The preference shock is assumed to follow the stationary autoregressive process

$$u_t^g = \rho_g u_{t-1}^g + \varepsilon_t^g, \qquad \varepsilon_t^g \sim i.i.d.N(0, \sigma_g^2). \tag{2}$$

Letting e_t denote the nominal exchange rate and p_t^* the foreign price level, the real exchange rate q_t is given by

$$q_t = e_t + p_t^* - p_t. \tag{3}$$

The household's choice between purchasing domestic or foreign bonds then implies the real interest rate parity condition

$$\left[r_t - \mathrm{E}_t\pi_{t+1}\right] - \left[r_t^* - \mathrm{E}_t\pi_{t+1}^*\right] = \mathrm{E}_t\Delta q_{t+1} - \chi a_t - u_t^q, \tag{4}$$

where r_t^* and π_t^* are the one-period nominal interest rate and the inflation rate in the foreign economy, a_t is the domestic economy's net foreign asset position, χ is the elasticity of the foreign exchange risk premium to the net foreign asset position, and u_t^q is a risk premium shock, assumed to follow

$$u_t^q = \rho_q u_{t-1}^q + \varepsilon_t^q, \qquad \varepsilon_t^q \sim i.i.d.N(0, \sigma_q^2). \tag{5}$$

The net foreign asset position, in turn, follows

$$a_t = \frac{1}{\beta}a_{t-1} + y_t - c_t - \alpha\left[e_t + p_t^* - p_t^f\right], \tag{6}$$

where β is the household's discount factor and α is the fraction of imported goods in the household's consumption basket.

There is a continuum of domestic firms producing differentiated goods under monopolistic competition using labor as the only input. These firms set prices in a staggered fashion, following Calvo (1983), so only a fraction $1-\theta_d$ of firms reset their prices optimally in each period. The remaining fraction partially index their prices to the previous period's inflation rate with indexation parameter δ_d. The rate of inflation in the domestic goods sector then follows

$$\begin{aligned}\pi_t^d &= \frac{\beta}{1+\beta\delta_d}\mathrm{E}_t\pi_{t+1}^d + \frac{\delta_d}{1+\beta\delta_d}\pi_{t-1}^d + \frac{(1-\theta_d)(1-\beta\theta_d)}{\theta_d(1+\beta\delta_d)}\mu_t + \varepsilon_t^{\pi d}, \\ \varepsilon_t^{\pi d} &\sim i.i.d.N(0,\sigma_{\pi d}^2),\end{aligned} \tag{7}$$

where μ_t is real marginal cost, and $\varepsilon_t^{\pi d}$ is a shock to firms' markup over marginal cost.[1] Combining the expression for marginal cost in the domestic sector with the optimal labor supply decision gives

$$\begin{aligned}\mu_t &= w_t - p_t^d - u_t^a \\ &= \varphi y_t - (1+\varphi)u_t^a + \alpha s_t + \frac{\sigma}{1-h}\left[c_t - hc_{t-1}\right],\end{aligned} \tag{8}$$

where w_t is the nominal wage, p_t^d is the price of domestic goods, u_t^a is a stationary technology shock that follows

$$u_t^a = \rho_a u_{t-1}^a + \varepsilon_t^a, \qquad \varepsilon_t^a \sim i.i.d.N(0,\sigma_a^2), \tag{9}$$

s_t is the terms of trade, defined as

$$s_t = p_t^f - p_t^d, \tag{10}$$

and φ is the inverse elasticity of labor supply.

There is also a continuum of firms importing goods from abroad under monopolistic competition. Marginal cost in the import sector is simply the domestic currency price of foreign goods, $e_t + p_t^*$, but the pricing power of import firms leads to short-run deviations from the law of one price, so $p_t^f \neq e_t + p_t^*$. As in the domestic sector, import firms also set prices in a staggered fashion, but with Calvo parameter θ_f and indexation parameter δ_f. Inflation in imported goods sector then follows

$$\pi_t^f = \frac{\beta}{1+\beta\delta_f}\mathrm{E}_t\pi_{t+1}^f + \frac{\delta_f}{1+\beta\delta_f}\pi_{t-1}^f + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\beta\delta_f)}\psi_t + u_t^{\pi f}, \tag{11}$$

where ψ_t is the deviation from the law of one price, given by

$$\psi_t = e_t + p_t^* - p_t^f, \tag{12}$$

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[1] This markup shock is not included in the original model by Justiniano and Preston (2008). However, as such a shock has important implications for monetary policy in a closed economy, and we want to compare the closed-economy policy implications to the open economy, we choose to include this shock in our model.

and $u_t^{\pi f}$ is a shock to the markup of import prices over marginal cost, assumed to follow

$$u_t^{\pi f} = \rho_{\pi f} u_{t-1}^{\pi f} + \varepsilon_t^{\pi f}, \qquad \varepsilon_t^{\pi f} \sim i.i.d.N(0, \sigma_{\pi f}^2). \tag{13}$$

We define the CPI inflation rate as

$$\begin{aligned} \pi_t &= (1-\alpha)\pi_t^d + \alpha \pi_t^f \\ &= \pi_t^d + \alpha \Delta s_t. \end{aligned} \tag{14}$$

We can then write the law-of-one-price gap ψ_t as

$$\psi_t = q_t - (1-\alpha) s_t. \tag{15}$$

Market clearing implies that domestic output is determined by

$$y_t = (1-\alpha)c_t + \alpha\eta(2-\alpha)s_t + \alpha\eta\psi_t + \alpha y_t^*, \tag{16}$$

where y_t^* is output in the foreign economy, and η is the elasticity of substitution between domestic and imported goods.

And finally, as the economy is small, the foreign economy (foreign inflation, output, and interest rate) is modelled as an exogenous vector autoregression with two lags:

$$\begin{bmatrix} \pi_t^* \\ y_t^* \\ r_t^* \end{bmatrix} = \sum_{j=1}^{2} \mathbf{B}_j \begin{bmatrix} \pi_{t-j}^* \\ y_{t-j}^* \\ r_{t-j}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\pi *} \\ \varepsilon_t^{y *} \\ \varepsilon_t^{r *} \end{bmatrix}, \tag{17}$$

where the shocks $\varepsilon_t^{\pi *}$, $\varepsilon_t^{y *}$, $\varepsilon_t^{r *}$ are i.i.d. normal with zero mean and variance $\sigma_{\pi *}^2, \sigma_{y *}^2, \sigma_{r *}^2$.

To parameterize the model, we use the estimates obtained by Justiniano and Preston (2008) using quarterly Australian data from 1984:I to 2007:I. For the foreign economy, they use U.S. data for the same period. These parameter estimates are shown in Tables 1–2.[2] When estimating the model, Justiniano and Preston assume that monetary policy follows a Taylor-type rule, that includes CPI inflation, the level and growth rate of domestic output, and the rate of nominal exchange rate depreciation. We will instead assume that the central bank sets monetary policy to minimize a quadratic loss function.

Viewed as a system, two features of the model are worth highlighting. First, the model does not allow a permanent trade-off between inflation and output, a knife-edge result that could easily be overturned if either equation (7) or equation (11) were misspecified. Second, it is movements in the law-of-one-price gap that are critical for output and inflation, not

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[2] We are grateful to Alejandro Justiniano and Bruce Preston for providing the exact parameter values. For the domestic markup shock, which was not included by Justiniano and Preston, we rely on an estimated standard deviation taken from Adolfson, Laséen, Lindé, and Villani (2008) using Swedish data.

movements in either the real exchange rate or the terms of trade. As a consequence, the model, as it stands, does not uniquely pin down steady-state values for either the real exchange rate or the terms of trade (the UIP condition has important implications for the change in the real exchange rate, but not for its level). Similarly, equation (15) shows that many combinations of the real exchange rate and the terms of trade are consistent with any given value of the law-of-one-price gap variable. Therefore, depending on how monetary policy is conducted, transitory shocks can have permanent effects on the real exchange rate and the terms of trade.

3 The robust control algorithm

When designing monetary policy, the central bank is assumed to use the estimated model in equations (1)–(17) as its "reference model," the model it believes best describes the data-generating process. However, the central bank fears that this reference model is misspecified, and therefore uses robust control methods to formulate monetary policy. As emphasized by Hansen and Sargent (2008), robust control allows the central bank to design a policy that guards purposefully against specification errors, or distortions, to the reference model that are "small" in the sense that the distorted model lies in a neighborhood "close" to the reference model. In formulating the central bank's robust control problem, we deviate slightly from Hansen and Sargent (2008) and allow the central bank to fear misspecification of both the conditional mean and the conditional volatility of the shock processes. Alternatively, our setup can be interpreted as the situation where the central bank sets policy before observing the shocks, while in the Hansen-Sargent setup, the central bank sets policy after observing the shocks.

Our robust control algorithms build on Dennis (2007) and Dennis, Leitemo, and Söderström (2008). These algorithms allow the optimization constraints to be written in a structural form as

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathrm{E}_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\boldsymbol{\varepsilon}_t, \tag{18}$$

where $\mathbf{y}_t$ is a vector of endogenous variables, $\mathbf{u}_t$ is a vector of policy instrument(s), $\mathbf{v}_t$ is a vector of specification errors, $\boldsymbol{\varepsilon}_t$ is a vector of innovations, and $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, and $\mathbf{A}_4$ are matrices conformable with $\mathbf{y}_t$, $\mathbf{u}_t$, and $\boldsymbol{\varepsilon}_t$ that contain the parameters of the model. The matrix $\mathbf{A}_0$ is assumed to be nonsingular and the elements of $\mathbf{A}_4$ are determined to ensure that the shocks are distributed according to $\boldsymbol{\varepsilon}_t \sim i.i.d.\,[\mathbf{0}, \mathbf{I}]$. The dating convention is such that any variable that enters $\mathbf{y}_{t-1}$ is predetermined, known by the beginning of period t.

Following Hansen and Sargent (2008), the central bank's fear of misspecification is formalized by introducing specification errors to each equation in which there is a shock. To help it devise a robust policy, the central bank assumes that where it desires to minimize a loss function, a fictitious "evil agent" strategically chooses the specification errors to maximize the loss function. To obtain the distorted model, we first introduce the expectational errors, $\boldsymbol{\varepsilon}_{\mathbf{y}t+1} \equiv \mathbf{y}_{t+1} - \mathrm{E}_t \mathbf{y}_{t+1}$, which will be a linear function of the innovations in equilibrium, $\boldsymbol{\varepsilon}_{\mathbf{y}t+1} = \mathbf{C}\boldsymbol{\varepsilon}_{t+1}$, and write equation (18) in terms of realizations as

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\boldsymbol{\varepsilon}_t - \mathbf{A}_2\mathbf{C}\boldsymbol{\varepsilon}_{t+1}, \tag{19}$$

where the matrix $\mathbf{C}$ has yet to be determined. Next, equation (19) is surrounded with a class of distorted models of the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\left(\mathbf{v}_t + \boldsymbol{\varepsilon}_t\right) - \mathbf{A}_2\mathbf{C}\left(\mathbf{v}_{t+1} + \boldsymbol{\varepsilon}_{t+1}\right), \tag{20}$$

where the sequence of specification errors, $\{\mathbf{v}_t\}$, is constrained to satisfy

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\mathbf{v}_t'\mathbf{v}_t \leq \eta, \tag{21}$$

where $\eta \in [0, \overline{\eta})$ represents the total "budget" for misspecification.

The central bank's loss function is assumed to take the form

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t\right], \tag{22}$$

where $\mathbf{W}$ and $\mathbf{Q}$ contain policy weights and are assumed to be symmetric positive-semidefinite and symmetric positive-definite, respectively. The parameter $\beta \in (0,1)$ is the central bank's discount factor.

Hansen and Sargent (2008) show that the problem of minimizing equation (22) with respect to $\mathbf{u}_t$ and maximizing with respect to $\mathbf{v}_t$ subject to equations (20) and (21) can be replaced with an equivalent multiplier problem in which

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t - \theta\mathbf{v}_t'\mathbf{v}_t\right], \tag{23}$$

is minimized with respect to $\mathbf{u}_t$ and maximized with respect to $\mathbf{v}_t$, subject to equation (20). The parameter $\theta \in (\underline{\theta}, \infty]$ represents the shadow price of a marginal relaxation of the constraint in equation (21) and is inversely related to the budget for misspecification, η.

Given a conjecture of $\mathbf{C}$, the Lagrangian for the robust decision problem is

$$\begin{aligned} L = \mathrm{E}_0\sum_{t=0}^{\infty}\beta^t \Bigg\{ & \mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{R}\mathbf{u}_t - \theta\mathbf{v}_t'\mathbf{v}_t \\ & +2\boldsymbol{\lambda}_t\Big[\mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\left(\mathbf{v}_t + \boldsymbol{\varepsilon}_t\right) \\ & -\mathbf{A}_2\mathbf{C}\left(\mathbf{v}_{t+1} + \boldsymbol{\varepsilon}_{t+1}\right) - \mathbf{A}_0\mathbf{y}_t\Big] \Bigg\}, \end{aligned} \tag{24}$$

where the vector $\boldsymbol{\lambda}_t$ contains the Lagrange multipliers on the distorted model.

The first order conditions of the Lagrangian with respect to $\boldsymbol{\lambda}_t$, $\mathbf{y}_t$, $\mathbf{u}_t$, and $\mathbf{v}_t$ are

$$\frac{\partial L}{\partial \boldsymbol{\lambda}_t} : \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathrm{E}_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\left(\mathbf{v}_t + \boldsymbol{\varepsilon}_t\right) - \mathbf{A}_2\mathbf{C}\mathrm{E}_t\mathbf{v}_{t+1} - \mathbf{A}_0\mathbf{y}_t = \mathbf{0}, \tag{25}$$

$$\frac{\partial L}{\partial \mathbf{y}_t} : \mathbf{W}\mathbf{y}_t + \beta\mathbf{A}_1'\mathrm{E}_t\boldsymbol{\lambda}_{t+1} + \beta^{-1}\mathbf{A}_2'\boldsymbol{\lambda}_{t-1} - \mathbf{A}_0'\boldsymbol{\lambda}_t = \mathbf{0}, \tag{26}$$

$$\frac{\partial L}{\partial \mathbf{u}_t} : \mathbf{R}\mathbf{u}_t + \mathbf{A}_3'\boldsymbol{\lambda}_t = \mathbf{0}, \tag{27}$$

$$\frac{\partial L}{\partial \mathbf{v}_t} : -\theta\mathbf{v}_t + \mathbf{A}_4'\boldsymbol{\lambda}_t - \beta^{-1}\left(\mathbf{A}_2\mathbf{C}\right)'\boldsymbol{\lambda}_{t-1} = \mathbf{0}. \tag{28}$$

Solving equations (25) through (28) yields the solution

$$\boldsymbol{\lambda}_t = \mathbf{M}^W_{\boldsymbol{\lambda}\boldsymbol{\lambda}}\boldsymbol{\lambda}_{t-1} + \mathbf{M}^W_{\boldsymbol{\lambda}\mathbf{y}}\mathbf{y}_{t-1} + \mathbf{N}^W_{\boldsymbol{\lambda}}\boldsymbol{\varepsilon}_t, \tag{29}$$

$$\mathbf{y}_t = \mathbf{M}^W_{\mathbf{y}\boldsymbol{\lambda}}\boldsymbol{\lambda}_{t-1} + \mathbf{M}^W_{\mathbf{y}\mathbf{y}}\mathbf{y}_{t-1} + \mathbf{N}^W_{\mathbf{y}}\boldsymbol{\varepsilon}_t, \tag{30}$$

$$\mathbf{u}_t = \mathbf{F}^W_{\boldsymbol{\lambda}}\boldsymbol{\lambda}_{t-1} + \mathbf{F}^W_{\mathbf{y}}\mathbf{y}_{t-1} + \mathbf{F}^W_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_t, \tag{31}$$

$$\mathbf{v}_t = \mathbf{K}^W_{\boldsymbol{\lambda}}\boldsymbol{\lambda}_{t-1} + \mathbf{K}^W_{\mathbf{y}}\mathbf{y}_{t-1} + \mathbf{K}^W_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_t. \tag{32}$$

The solution to this robust control problem yields the central bank's "worst-case" equilibrium, the equilibrium in which the worst-case specification errors are realized, the central bank employs its robust decision rule, and private agents form expectations acknowledging the central bank's fear of misspecification. Once the worst-case equilibrium has been obtained, it is straightforward to obtain the "approximating" equilibrium, in which the central bank employs its robust decision rule and private agents form expectations acknowledging the central bank's fear of misspecification, but the reference model transpires to be specified correctly.

To obtain the worst-case equilibrium we update $\mathbf{C}$ according to $\mathbf{C} \leftarrow \mathbf{N}^W_{\mathbf{y}}$ and iterate over equations (25) through (32) until a fix-point is reached. Letting $\mathbf{z}_t \equiv [\ \boldsymbol{\lambda}_t' \quad \mathbf{y}_t'\ ]'$, the worst-case equilibrium can be written as

$$\mathbf{z}_t = \mathbf{M}^W\mathbf{z}_{t-1} + \mathbf{N}^W\boldsymbol{\varepsilon}_t, \tag{33}$$

$$\mathbf{u}_t = \mathbf{F}_{\mathbf{z}}\mathbf{z}_{t-1} + \mathbf{F}_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_t, \tag{34}$$

$$\mathbf{v}_t = \mathbf{K}_{\mathbf{z}}\mathbf{z}_{t-1} + \mathbf{K}_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_t. \tag{35}$$

The approximating equilibrium, which has the form,

$$\mathbf{z}_t = \mathbf{M}^A\mathbf{z}_{t-1} + \mathbf{N}^A\boldsymbol{\varepsilon}_t, \tag{36}$$

$$\mathbf{u}_t = \mathbf{F}_{\mathbf{z}}\mathbf{z}_{t-1} + \mathbf{F}_{\boldsymbol{\varepsilon}}\boldsymbol{\varepsilon}_t, \tag{37}$$

is then obtained by solving equation (18) jointly with equations (28) and (31).

Following Hansen and Sargent (2008), we determine the set of admissible specification errors by selecting the central bank's preference for robustness to generate a particular "detection error probability," the probability that an econometrician would infer incorrectly whether the approximating equilibrium or the worst-case equilibrium generated the observed data. The intuitive connection between θ and the probability of making a detection error is that when θ is small, greater differences between the distorted model and the reference model (more severe misspecifications) can arise, which are more easily detected. Let model A denote the approximating model and model W denote the worst-case model. Then the probability of making a detection error is given by

$$p(\theta) = \frac{\text{prob}\,(A|W) + \text{prob}(W|A)}{2}, \tag{38}$$

where prob$(A|W)$ (prob$(W|A)$) represents the probability that the econometrician erroneously chooses model A (model W) when in fact model W (model A) generated the data.

To calculate the detection error probability for a given θ, we assume that the selection of one model over another is based on the likelihood ratio principle. Therefore, with $\{\mathbf{z}_t^W\}_1^T$ denoting a finite sequence of economic outcomes generated by the worst-case equilibrium, model W, and L_{AW} and L_{WW} denoting the likelihood associated with models A and W, respectively, then the econometrician chooses model A over model W if $\log(L_{WW}/L_{AW}) < 0$. Generating M independent sequences $\{\mathbf{z}_t^W\}_1^T$, prob$\,(A|W)$ can be calculated according to

$$\text{prob}\,(A|W) \approx \frac{1}{M}\sum_{m=1}^{M} \text{I}\left[\log\left(\frac{L_{WW}^m}{L_{AW}^m}\right) < 0\right], \tag{39}$$

where $\text{I}[\log(L_{WW}^m/L_{AW}^m) < 0]$ is an indicator function that equals one when its argument is satisfied and equals zero otherwise; prob$(W|A)$ is calculated analogously using draws generated from the approximating model. The likelihood function that is generally used to calculate prob$(A|W)$ and prob$(W|A)$ assumes that the innovations are normally distributed.

To calculate detection error probabilities while accounting for the distortions to both the conditional means and the conditional volatilities of the shocks, let

$$\mathbf{z}_t^A = \mathbf{M}^A\mathbf{z}_{t-1} + \mathbf{N}^A\boldsymbol{\varepsilon}_t, \tag{40}$$

$$\mathbf{z}_t^W = \mathbf{M}^W\mathbf{z}_{t-1} + \mathbf{N}^W\boldsymbol{\varepsilon}_t \tag{41}$$

govern equilibrium outcomes under the approximating equilibrium and the worst-case equilibrium, respectively. When $\mathbf{N}^A \neq \mathbf{N}^W$, to calculate $p(\theta)$ we must first allow for the stochastic singularity that generally characterizes equilibrium and second account appropriately for the Jacobian of transformation that enters the likelihood function. Using the QR decomposition, we decompose $\mathbf{N}^A$ according to $\mathbf{N}^A = \mathbf{Q}_A\mathbf{R}_A$ and $\mathbf{N}^W$ according to $\mathbf{N}^W = \mathbf{Q}_W\mathbf{R}_W$.

By construction, $\mathbf{Q}_A$ and $\mathbf{Q}_W$ are orthogonal matrices ($\mathbf{Q}'_A\mathbf{Q}_A = \mathbf{Q}'_W\mathbf{Q}_W = \mathbf{I}$) and $\mathbf{R}_A$ and $\mathbf{R}_W$ are upper triangular. Let

$$\widehat{\boldsymbol{\varepsilon}}_t^{i|j} = \mathbf{R}_i^{-1}\mathbf{Q}'_i\left(\mathbf{z}_t^j - \mathbf{M}^i\mathbf{z}_{t-1}^j\right), \quad \{i,j\} \in \{A,W\} \tag{42}$$

represent the inferred innovations in period t when model i is fitted to data $\{\mathbf{z}_t^j\}_1^T$ that are generated according to model j and let $\widehat{\boldsymbol{\Sigma}}^{i|j}$ be the associated estimates of the innovation variance-covariance matrices. Then

$$\log\left(\frac{L_{AA}}{L_{WA}}\right) = \log\left|\mathbf{R}_A^{-1}\right| - \log\left|\mathbf{R}_W^{-1}\right| + \frac{1}{2}\mathrm{tr}\left(\widehat{\boldsymbol{\Sigma}}^{W|A} - \widehat{\boldsymbol{\Sigma}}^{A|A}\right), \tag{43}$$

$$\log\left(\frac{L_{WW}}{L_{AW}}\right) = \log\left|\mathbf{R}_W^{-1}\right| - \log\left|\mathbf{R}_A^{-1}\right| + \frac{1}{2}\mathrm{tr}\left(\widehat{\boldsymbol{\Sigma}}^{A|W} - \widehat{\boldsymbol{\Sigma}}^{W|W}\right), \tag{44}$$

where "tr" is the trace operator.

Given equations (43) and (44), equation (39) is used to estimate prob$(A|W)$ and (similarly) prob$(W|A)$, which are needed to construct the detection error probability, as per equation (38). The multiplier, θ, is then determined by selecting a detection error probability (or at least its lower bound) and inverting equation (38). Generally this inversion is performed numerically by constructing the mapping between θ and the detection error probability, for a given sample size.

4 Robust monetary policy

We now study the properties of robust monetary policy in our model of the Australian economy. We assume that the central bank's goals are to stabilize four-quarter CPI inflation, $\bar{\pi}_t \equiv \sum_{j=0}^{3}\pi_{t-j}$; the level of output, y_t; and the annualized quarterly interest rate, $\tilde{r}_t \equiv 4r_t$, around their long-run steady-state levels. The central bank's objectives are summarized by the quadratic loss function

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left[\bar{\pi}_t^2 + \lambda y_t^2 + \nu\tilde{r}_t^2\right], \tag{45}$$

where we set $\beta = 0.99$, $\lambda = 0.5$, and $\nu = 0.1$. These weights imply that the economy under the non-robust policy displays fluctuations similar to the data used for estimation.[3]

We focus on the case where monetary policy and the specification errors are chosen with commitment. We then apply our robust control algorithm to construct the robust monetary policy that guards against distortions to the reference model described by equations (1)–(17).

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[3] More specifically, in Australian data from 1984:I to 2007:I, the standard deviations of annualized quarterly inflation, detrended GDP, the rate of real exchange rate depreciation and the short-term interest rate are, respectively, 2.73, 1.98, 4.72, and 1.09 percentage points. In the model with the optimal non-robust policy with commitment, these standard deviations are 2.00, 1.48, 4.64, and 1.14.

To isolate the effects of the transmission channels/shocks that are specific to the open economy, we first analyze a "pseudo-closed" version of the model, eliminating all open-economy elements by setting the open-economy parameters and shocks to zero. This exercise establishes the effects of robust monetary policy in a closed economy, providing a benchmark against which to compare the open-economy results. We then proceed by systematically adding open-economy elements to the reference model.

For each specification, we compare the outcomes of the rational expectations equilibrium (RE), the worst-case equilibrium (WO), and the approximating equilibrium (AP). Throughout, we choose the central bank's preference for robustness so that the detection error probability equals 0.2, calculated using $1,000$ simulated samples of 200 observations. This detection error probability allows the distortions to the reference model to be of a reasonable magnitude, but not so large as to make it inconceivable that they would not have been detected previously.

4.1 Robust monetary policy in a "pseudo-closed" economy

We first analyze the "pseudo-closed" version of our model. To do this, we shut down all open-economy transmission channels and shocks, leaving the three-equation system

$$y_t = \frac{1}{1+h}\mathrm{E}_t y_{t+1} + \frac{h}{1+h} y_{t-1} - \frac{1-h}{\sigma(1+h)}\left[r_t - \mathrm{E}_t\pi^d_{t+1} - u^g_t + \mathrm{E}_t u^g_{t+1}\right], \tag{46}$$

$$\pi^d_t = \frac{\beta}{1+\beta\delta_d}\mathrm{E}_t\pi^d_{t+1} + \frac{\delta_d}{1+\beta\delta_d}\pi^d_{t-1} + \frac{(1-\theta_d)(1-\beta\theta_d)}{\theta_d(1+\beta\delta_d)}\mu_t + \varepsilon^{\pi d}_t, \tag{47}$$

$$\mu_t = \varphi y_t - (1+\varphi)u^a_t + \frac{\sigma}{1-h}\left[y_t - h y_{t-1}\right], \tag{48}$$

where we have used the fact that $y_t = c_t$ in the closed economy.

Figure 1 shows how key variables in the model respond to impulses to the three shocks: to technology, consumer preferences, and the markup of domestic prices over marginal cost. Consider first the responses under the non-robust policy (or rational expectations), represented by the solid lines. A positive technology shock lowers marginal cost and inflation, and at the same time increases output. As a response, monetary policy is first tightened to reduce output, and then expanded to offset the fall in inflation. A positive preference shock raises consumption and output, which increases marginal cost and therefore inflation. The central bank therefore tightens policy, and output, marginal cost and inflation return to steady state after a period of overshooting. While the preference shock has very small effects on the economy, the impact of the technology shock is substantially larger. The technology shock does not, however, create a serious tradeoff for the central bank, as it tends to move output and inflation in opposite directions, which over time act to offset each other. In contrast, the third shock, the price markup shock, has large effects on the economy and creates an important policy tradeoff. A positive markup shock increases inflation, forcing the central bank to reduce output and marginal cost by raising the interest rate. Inflation then falls back toward steady state with some overshooting.

When we introduce a preference for robustness, the central bank typically fears that the economy will fluctuate more in response to the shocks, as well as to the policy response. For the consumption preference shock, the effects of robustness are not great, as this shock already has a small impact on the economy. Following a technology shock, on the other hand, the robust central bank fears very large movements in output, marginal cost, and inflation, and responds by a much more aggressive movements in the interest rate. Following a price markup shock, the central bank fears that the impact on inflation will be larger than in the reference model, and responds with a more aggressive policy tightening, which leads to larger declines in output and marginal costs.

Panel (a) of Table 3 reports the unconditional standard deviations of key variables and the value of the loss function under the non-robust and robust policies. Overall, the robust central bank fears that inflation and output will be much more volatile than they are in the reference model, leading to more volatilty also in the interest rate. With the robust policy (in the approximating equilibrium), the standard deviation of output is almost double that with rational expectations, and the volatility of inflation and the interest rate are also substantially higher. Under the robust policy, the value of the loss function almost doubles.

To illustrate the size of specification errors in the worst-case model, Panel (a) of Table 4 shows the variances of these errors and Table 5 shows the effects on the variances of the structural shocks. Since the price markup shock creates the most difficult tradeoff for the central bank, the distortions to this shock are considerably larger than those to the other two shocks. This is also illustrated by the distorted variances of the structural shocks, where there is a sizeable impact only on the variance of the price markup shock.

Thus, the robust central bank in this pseudo-closed economy should mainly worry about specification errors to the inflation equation. The cost of insuring against this misspecification comes in the form of greater volatility in the interest rate and output. These results are qualitatively similar to those reached by Dennis, Leitemo, and Söderström (2008), who examine a related closed-economy model, and by Leitemo and Söderström (2008a), who study a more stylized model. We now turn our attention to adding open-economy features to the model.

4.2 Introducing open-economy channels

We first introduce the open-economy transmission channels, but keep the domestic shocks as the only source of fluctuations. Accordingly, the reference model is given by equations (1)–

(17), but we shut down the shocks to the imported price markup ($\varepsilon_t^{\pi f}$), the foreign economy ($\varepsilon_t^{\pi *}, \varepsilon_t^{y*}, \varepsilon_t^{r*}$), and the foreign exchange risk premium (ε_t^{q}). In this specification, the three domestic shocks, as well as monetary policy interventions, have additional effects on the economy through imported-goods inflation and the real exchange rate.

Figures 2–3 show impulse responses to these three shocks, and Panel (b) of Tables 3–5 show the corresponding results on overall volatility in the model.

In general, the impulse responses for the non-robust policy reveal that the central bank actively uses the open-economy transmission channels to stabilize the economy. For instance, after a technology shock, the central bank lowers the interest rate, leading to a real exchange rate depreciation and higher import-price inflation. Similarly, after a consumption preference shock, the higher interest rate leads to a real exchange rate appreciation, which reduces import-price inflation and therefore offsets the impact of higher domestic-price inflation on the consumer price index. As monetary policy in the open economy has a more powerful impact than in a closed economy, the central bank can be less active in its interest rate adjustments in response to these shocks.

Following a price markup shock, the open-economy features instead serve to make the central bank behave more aggressively. The optimal policy is to raise interest rates to reduce output and marginal costs. But the real exchange rate appreciation implies that a given interest rate increase has a smaller impact on consumption and output and, as a consequence, the central bank needs to tighten policy more aggressively to stabilize inflation.

Overall, when the central bank is able to exploit the open-economy transmission channels, it is able to better stabilize the economy after shocks. Therefore, with the non-robust policy, output and inflation are more stable than in the closed economy, and loss is about 50 percent lower, see Table 3. Central bank robustness against model misspecification has similar effects to those in the closed economy, although the central bank now also fears that the exchange rate may be more volatile than the reference model would suggest. When the central bank is robust, as in the closed economy, it fears that inflation and output are more volatile causing it to respond more aggressively to shocks. But the open-economy channels also help the central bank counteract misspecification, so the specification errors are less damaging than in the closed economy: in the approximating model, loss is 60 percent higher than with rational expectations, compared to an almost doubling in loss in the closed economy. This increase in loss is largely due to a rise in output volatility, with small effects from CPI inflation and the interest rate.

Relative to the pseudo-closed economy, the main implications for robust monetary policy remain largely unaltered. The central bank continues to fear that shocks will have larger and more persistent effects on domestic inflation than they do in the reference model. As we will see next, however, introducing the open-economy shocks creates new sources of specification errors and has a substantial impact on the robust monetary policy.

4.3 The influence of import price markup shocks

We next introduce the import price markup shock. Figures 4–5 show the impulse responses following an import price markup shock and, for comparison, the equivalent responses for a domestic markup shock. Of course, under the non-robust policy, the response to the domestic markup shock is identical to the case with only domestic shocks in Figure 2. But with the robust policy, the worst-case specification errors are different, as the "evil agent" will reallocate the distortions when there is a fourth shock in the model. (The robust responses to the preference shock and the technology shock are still very similar to the earlier case, so these are not shown.) Panel (c) of Tables 3–5 show the corresponding results on overall volatility in the model.

After a positive shock to the import price markup, imported inflation increases. To offset this impact on import price inflation (and therefore CPI inflation), the central bank needs to reduce the law-of-one-price gap. It achieves this by using tighter monetary policy to generate a real exchange rate appreciation. Since import prices do not adjust one-for-one with the real exchange rate, there will be a negative deviation from the law of one price, and over time, import price inflation will return to steady state (with a long period of overshooting). The tighter monetary policy also reduces output, but domestic price inflation increases, because a small improvement in the terms of trade pushes up marginal costs.

Under the robust policy, the central bank is highly concerned with distortions to the import price Phillips curve, making distortions to the domestic inflation equation less prominent. Following an import price markup shock, the central bank fears that the real exchange rate will appreciate much more strongly than in the reference model, so much as to reverse the effects of the shock on import price inflation. As a consequence, the central bank does not raise the interest rate as much as in the reference model, but instead initially lowers the interest rate before generating a modest tightening. The strong real exchange rate appreciation leads to a larger fall in output, but to an increase in domestic inflation, again due to movements in marginal cost. The overall effects of robustness on CPI inflation are however modest.

Panel (c) of Table 3 shows that the import price markup shock generates considerable volatility, with loss increasing by 75 percent relative to when there are only domestic shocks. Fears for model misspecification serve to increase the volatility of output and the real exchange rate, but again have only small effects on CPI inflation and the interest rate. Tables 4 and 5 reveal that the distortions to the two inflation equations are large, while the others are, as before, extremely small. The import price markup shock is thus responsible for a large part of the volatility of the small open economy, making the import price Phillips curve a key concern as a source of model misspecification.

4.4 The influence of foreign shocks

As a next step, we introduce the shocks originating in the foreign economy, continuing, however, to assume that there are no shocks to the interest parity condition. Our experiments show that the responses to the domestic shocks and the import price markup shock remain essentially unaltered. Consequently, Figures 6–7 show only the impulse responses to the foreign shocks.

Following a shock to foreign output, the foreign interest rate increases. As a consequence, domestic output, marginal costs, and domestic inflation all rise. In response, the central bank increases the interest rate, causing the real exchange rate to appreciate, which drives down import price inflation and eventually also CPI inflation.

After a foreign inflation shock, the foreign interest rate increases and foreign output falls. Facing lower foreign demand and higher foreign interest rates, domestic output falls and the real exchange rate depreciates. The exchange rate depreciation causes imported inflation and CPI inflation increase. The central bank tightens monetary policy, leading to even lower domestic output, marginal cost, and domestic inflation, which stabilizes CPI inflation.

Following a foreign interest rate shock, the real exchange rate depreciates causing domestic output and marginal costs to fall, while putting upward pressure on import price inflation. Again, the central bank needs to tighten monetary policy to reduce domestic inflation and offset the effects on CPI inflation.

Overall, the effects of foreign shocks on the domestic economy are modest and for this reason the robust central bank does not greatly fear distortions to this nexus of the model. Panel (d) of Tables 4 and 5 also show that there are essentially no distortions to the foreign equations and that the other distortions remain largely unaffected by the introduction of foreign shocks.

4.5 The complete open-economy model

Finally we add the foreign exchange risk premium shock, ε_t^q. Interestingly, introducing this shock has virtually no effects on the robust responses to the other shocks. For this reason, Figure 8 shows only the impulse responses to the risk premium shock.

A positive shock to the exchange rate risk premium leads to a large real appreciation, so import price inflation falls substantially, while marginal cost and domestic inflation increase. The central bank then needs to cut the interest rate to offset the real appreciation and increase CPI inflation. Somewhat surprisingly, introducing a preference for robustness has fairly small effects on the behavior of the model. The real exchange rate depreciates slightly more, with larger effects on import price inflation and domestic inflation. Therefore, the central bank needs to cut the interest rate more aggressively.

Table 3 shows that introducing the exchange rate shock leads to increased volatility in the real exchange rate, imported inflation, and interest rate, with small effects on CPI inflation and output. The fear of misspecification still has large effects on the volatility of the real exchange rate and output, and the robust policy causes loss to rise by some 60 percent relative to the non-robust policy. However, Tables 4 and 5 reveal that the worst-case specfication errors to the interest rate parity condition are one order of magnitude smaller than those to the two Phillips curves, and the conditional variance of the risk premium shock is hardly distorted at all. Thus, the additional volatility under the robust policy comes mainly from the fear of distortions to the Phillips curves rather than to the exchange rate.

5 Conclusion

We study the effects of model uncertainty on monetary policy in a small open-economy. We have done this incrementally, moving from a pseudo-closed economy model to an open economy model, adding structure at each step. Along the way we have demonstrated that a robust central bank in a closed economy fears mainly that inflation and output shocks will have larger and more persistent effects on inflation than they do in the reference model. Fearing this persistence, the robust central bank responds aggressively to shocks, giving rise to less inflation volatility but more output volatility than the non-robust policy.

We have also shown that the open-economy transmission channels *per se* do not have a large effect on the robust policy. If the only shocks in the economy are to domestic output and inflation, then the conclusions from the closed-economy model remain largely unaltered: the robust central bank fears mainly that the equation for domestic inflation might be misspecified, because distortions to the Phillips curve pose a difficult stabilization problem for the central bank. But the open-economy transmission channels help the central bank to stabilize the economy after shocks, lowering the volatility of all variables.

Introducing shocks to imported-goods price inflation adds significantly to the size of business cycle fluctuations. Adding shocks to the foreign economy or the foreign exchange risk premium, on the other hand, has modest effects. The robust central bank in the open economy therefore mainly fears misspecification in the relationships determining import-price inflation and domestic-price inflation, that is the Phillips curves in the import and domestic sectors.

These results suggest that understanding the nature of price setting and the impact of exchange rate movements on import prices (that is, the degree of exchange rate pass-through) should be a key concern for central banks in small open economies. It seems less crucial to understand the determination of the exchange rate itself, or the nature of deviations from uncovered interest rate parity.

The finding that deviations from uncovered interest rate parity are not very damaging, nor very vulnerable to model misspecification, depends partly on the assumption that monetary policy is set with commitment. The central bank then has considerable influence over private sector expectations, which helps it to control the exchange rate. Although full commitment may not be a perfectly realistic assumption, neither is full discretion. Many central banks in small open economies have explicit inflation targets and use very transparent monetary policy procedures. Many also publish forecasts of key variables, such as inflation, output growth, or even the short-term interest rate. These strategies have developed as a means to better anchor private expectations. To the extent that such strategies are successful in facilitating commitment on the part of the central bank, they may also allow central banks to be less concerned about deviations from interest rate parity.

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Table 1: Structural parameter values

Description	Notation	Value
Calibrated structural parameters		
Share of foreign goods in consumption	α	0.185
Discount factor	β	0.99
Elasticity of risk premium to net foreign assets	χ	0.01
Estimated structural parameters		
Inverse elasticity of intertemporal substitution	σ	1.309
Inverse elasticity of labor supply	φ	1.1157
Elasticity of substitution between domestic and imported goods	η	0.5824
Habit parameter	h	0.33
Domestic price Calvo parameter	θ_d	0.7935
Import price Calvo parameter	θ_f	0.5511
Domestic price indexation parameter	δ_d	0.0499
Import price indexation parameter	δ_f	0.0693
Shock persistence parameters		
Technology shock	ρ_a	0.6936
Preference shock	ρ_g	0.9257
Import price markup shock	$\rho_{\pi f}$	0.9352
Risk premium shock	ρ_q	0.9384
Shock standard deviations		
Technology shock	σ_a	0.3665
Preference shock	σ_g	0.1610
Domestic price markup shock	$\sigma_{\pi d}$	0.7690
Import price markup shock	$\sigma_{\pi f}$	1.5769
Risk premium shock	σ_q	0.3470

Note: This table shows parameters estimated by Justiniano and Preston (2008) on quarterly Australian data from 1984:I to 2007:I, except $\sigma_{\pi d}$ which is estimated by Adolfson, Laséen, Lindé, and Villani (2008) on quarterly Swedish data from 1993:I to 2005:III. The parameters are median values from the estimated posterior distribution.

Table 2: Parameter values for foreign economy VAR

Notation	Value		
VAR parameters			
$\mathbf{B}_1$	0.3242	0.0558	0.1308
	-0.1162	1.0378	0.1678
	0.0807	0.1098	1.1031
$\mathbf{B}_2$	-0.0078	-0.0359	-0.0364
	-0.0907	-0.1260	-0.0268
	0.0396	-0.1036	-0.2102
Shock standard deviations			
$\sigma_{\pi*}$	0.3498		
σ_{y*}	0.4795		
σ_{r*}	0.1151		

Note: This table shows parameters estimated by Justiniano and Preston (2008) on quarterly Australian data from 1984:I to 2007:I. The parameters are median values from the estimated posterior distribution.

Table 3: Unconditional standard deviations and loss in different versions of the model

	Standard deviation							Loss
	$\bar{\pi}_t$	$\tilde{\pi}_t$	$\tilde{\pi}_t^d$	$\tilde{\pi}_t^f$	y_t	Δq_t	$\tilde{r}_t$	
(*a*) *Closed-economy version*								
RE	1.150	2.675			0.888		0.381	1.679
WO	1.388	2.899			1.630		0.552	3.159
AP	1.388	2.900			1.618		0.551	3.141
(*b*) *Open-economy model with only domestic shocks*								
RE	0.722	1.906	2.443	0.729	0.776	0.833	0.510	0.825
WO	0.813	1.994	2.635	1.235	1.169	1.090	0.630	1.330
AP	0.814	1.994	2.636	1.235	1.162	1.090	0.630	1.324
(*c*) *Open-economy model with only domestic and import price markup shocks*								
RE	0.763	1.949	2.467	2.841	1.425	3.389	0.633	1.450
WO	0.803	1.983	2.617	2.623	3.890	5.027	0.682	4.490
AP	0.801	1.983	2.607	2.470	2.008	5.046	0.669	2.352
(*d*) *Open-economy model without exchange rate shock*								
RE	0.767	1.953	2.479	3.230	1.437	3.475	0.691	1.475
WO	0.808	1.989	2.632	3.142	3.951	5.105	0.737	4.602
AP	0.806	1.989	2.622	3.014	2.026	5.124	0.724	2.394
(*e*) *Open-economy model with all shocks*								
RE	0.808	2.002	2.669	6.679	1.475	4.642	1.139	1.650
WO	0.866	2.063	2.818	7.099	3.423	6.022	1.252	4.491
AP	0.864	2.064	2.809	7.057	2.087	6.039	1.210	2.653

Note: This table shows the unconditional standard deviations of key variables and expected loss in five versions of the open-economy model when monetary policy and specification errors are set with commitment. “RE” represents the outcome with rational expectations and non-robust monetary policy, “WO” is the outcome in the worst-case equilibrium with robust policy, “AP” is the outcome in the approximating equilibrium with robust policy. $\bar{\pi}_t$ is four-quarter inflation, $\tilde{\pi}_t^d$, $\tilde{\pi}_t^f$, $\tilde{r}_t$ are annualized quarterly domestic and import price inflation and one-period interest rate, respectively. The loss function is given by equation (45) with $\beta = 0.99$, $\lambda = 0.5$, and $\nu = 0.1$; the preference for robustness is chosen to produce a detection error probability of 0.2.

Table 4: Unconditional variances of specification errors

Specification error							
v_t^g	v_t^a	$v_t^{\pi d}$	$v_t^{\pi f}$	$v_t^{\pi *}$	$v_t^{y *}$	$v_t^{r *}$	v_t^q
(a) *Closed-economy version*							
1.3×10^{-7}	2.7×10^{-4}	0.013					
(b) *Open-economy model with only domestic shocks*							
2.3×10^{-7}	2.0×10^{-4}	0.012					
(c) *Open-economy model with only domestic and import price markup shocks*							
1.0×10^{-7}	7.6×10^{-4}	0.027	0.039				
(d) *Open-economy model without exchange rate shock*							
1.1×10^{-7}	0.8×10^{-4}	0.028	0.040	2.9×10^{-6}	1.5×10^{-4}	4.4×10^{-5}	
(e) *Open-economy model with all shocks*							
2.7×10^{-7}	5.3×10^{-4}	0.019	0.025	2.8×10^{-6}	7.2×10^{-5}	1.1×10^{-4}	2.9×10^{-3}

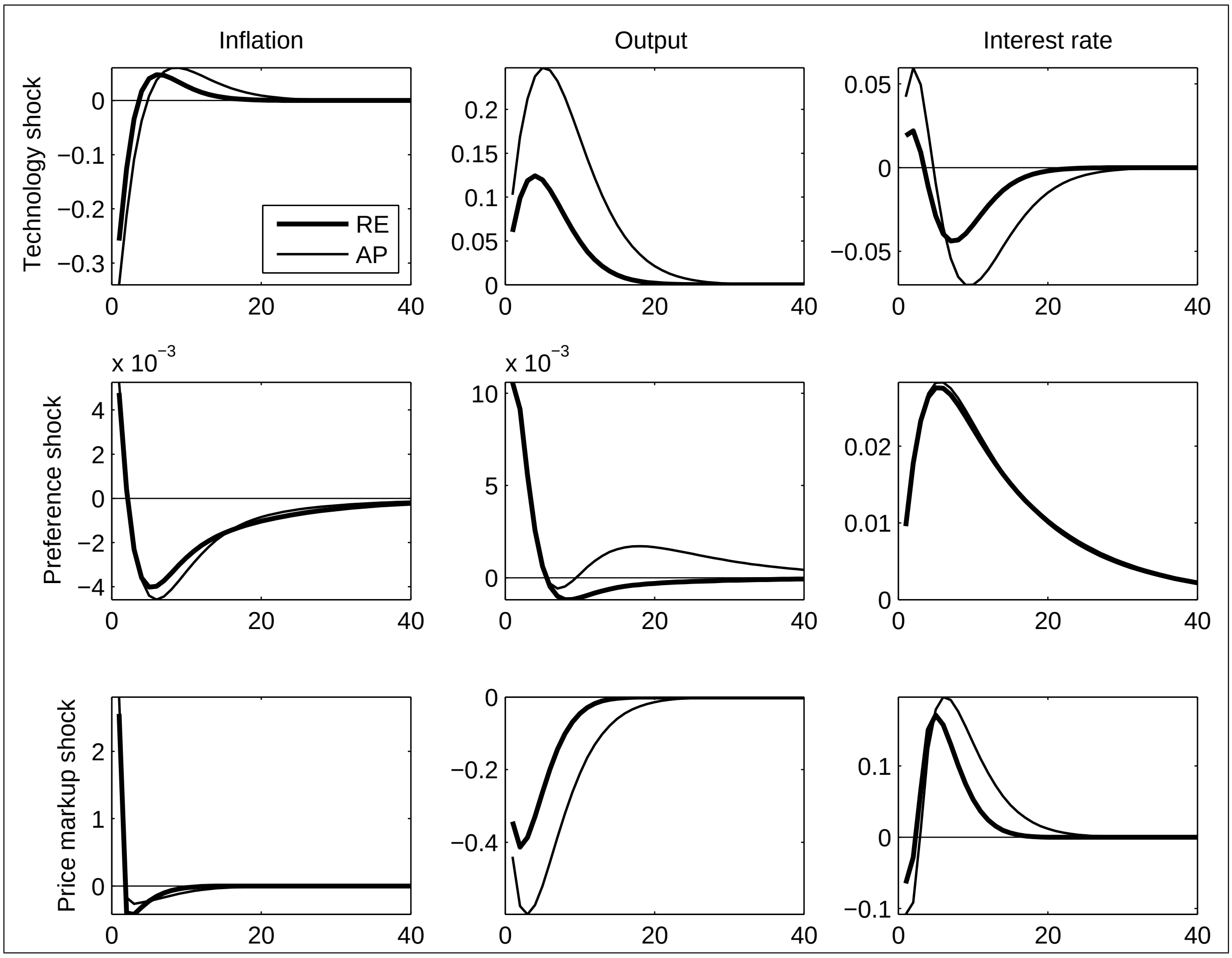
Note: This table shows the unconditional variances of worst-case specification errors in five versions of the open-economy model when monetary policy and specification errors are set with commitment. The preference for robustness is chosen to produce a detection error probability of 0.2.

Table 5: Distortions to conditional variances of structural shocks

Shock							
ε_t^g	ε_t^a	$\varepsilon_t^{\pi d}$	$\varepsilon_t^{\pi f}$	$\varepsilon_t^{\pi *}$	ε_t^{y*}	ε_t^{r*}	ε_t^q
Structural variances							
0.026	0.134	0.591	2.487	0.122	0.230	0.013	0.120
(a) *Closed-economy version*							
0.026	0.135	0.621					
(b) *Open-economy model with only domestic shocks*							
0.026	0.135	0.634					
(c) *Open-economy model with only domestic and import price markup shocks*							
0.026	0.134	0.605	2.572				
(d) *Open-economy model without exchange rate shock*							
0.026	0.134	0.605	2.573	0.122	0.230	0.013	
(e) *Open-economy model with all shocks*							
0.026	0.134	0.604	2.567	0.122	0.230	0.013	0.121

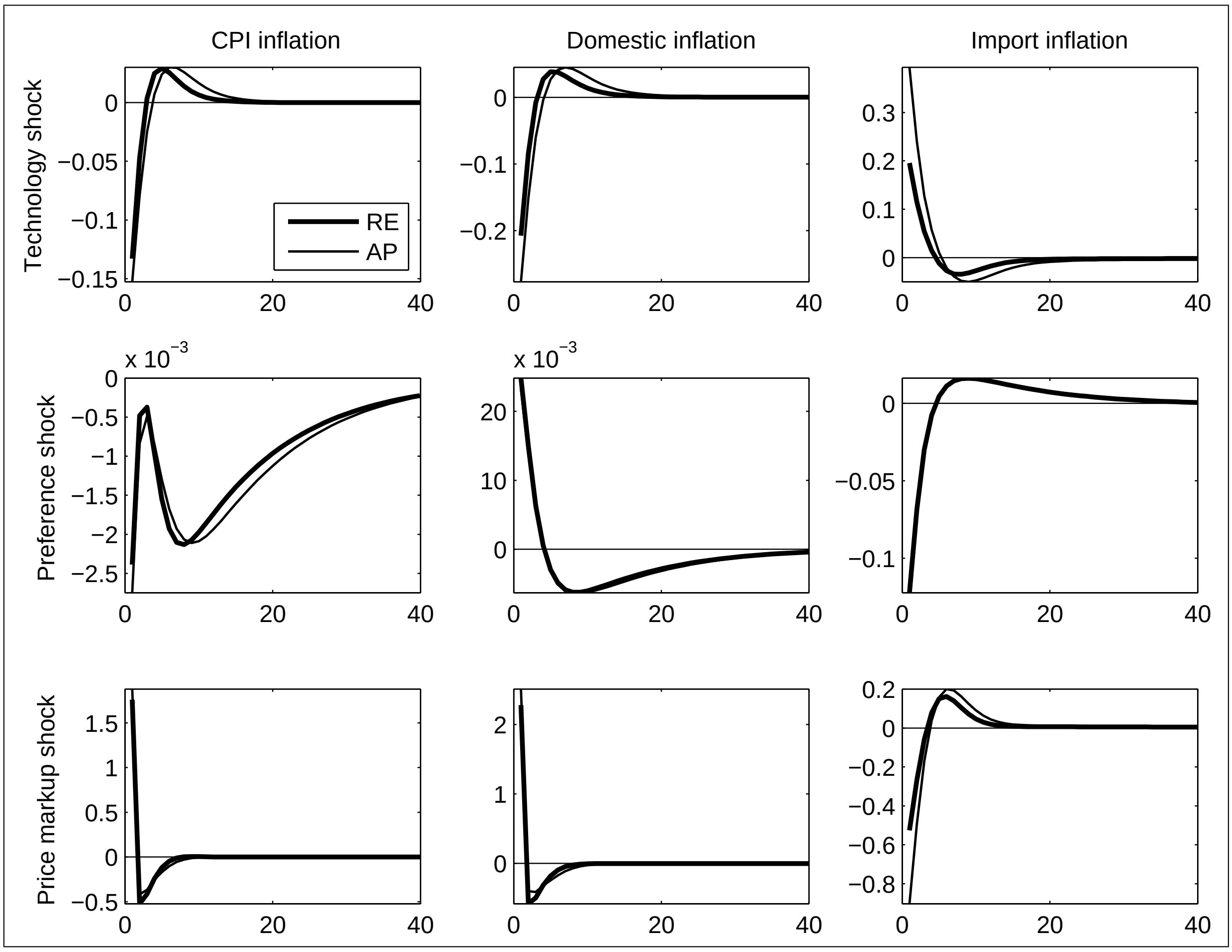
Note: This table shows the impact of worst-case specification errors on the variances of shocks in five versions of the open-economy model when monetary policy and specification errors are set with commitment. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 1: Impulse responses in closed-economy version of the model



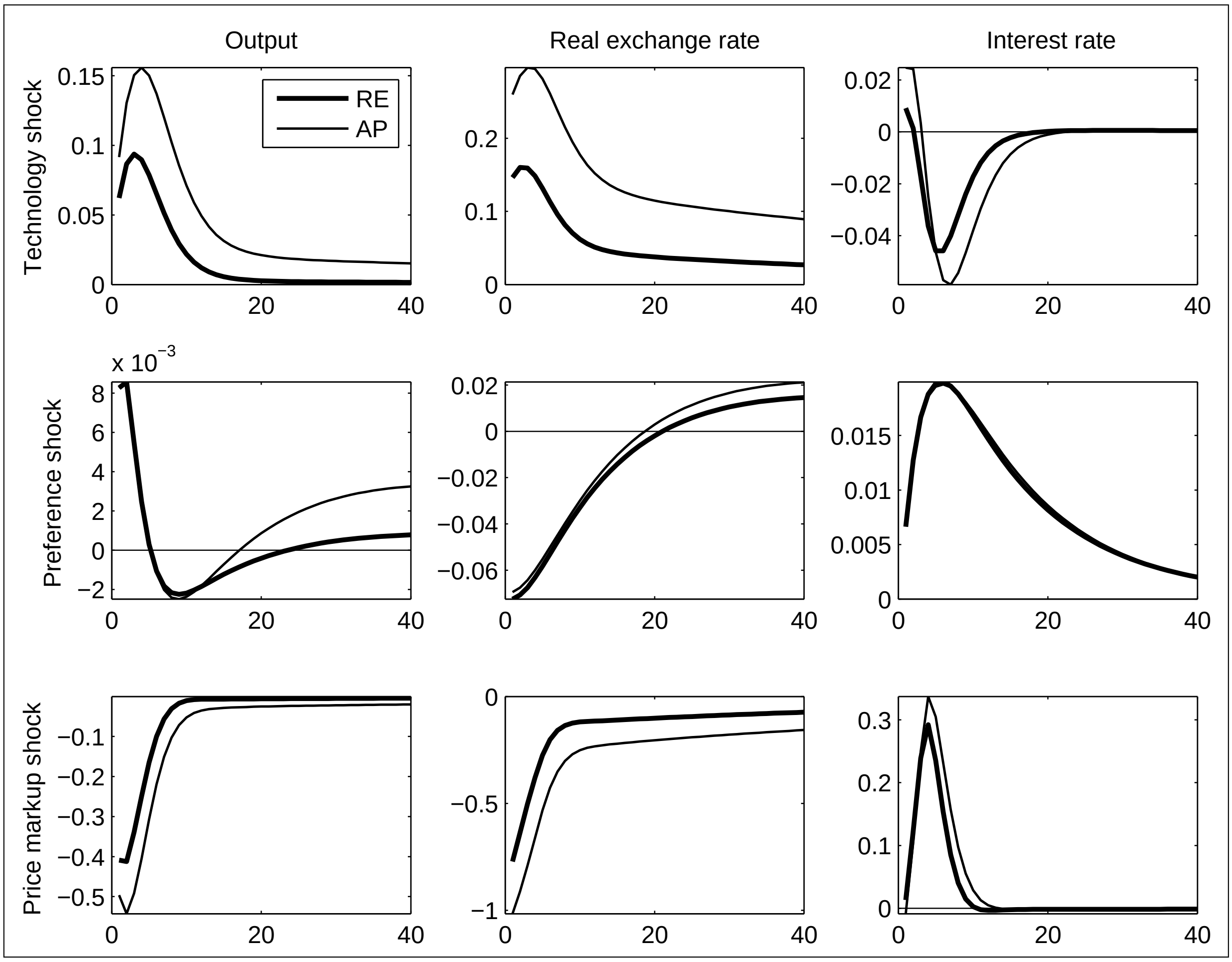
Note: The figure shows impulse responses of key variables to shocks (of one standard deviation) in the closed-economy version of the model when monetary policy and specification errors are set with commitment. “RE” represents the outcome with rational expectations and non-robust monetary policy, “AP” is the outcome in the approximating equilibrium with robust policy. The inflation rate is the annualized quarterly change in the consumer price level, the interest rate is expressed in annualized terms. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 2: Impulse responses in open-economy model with only domestic shocks



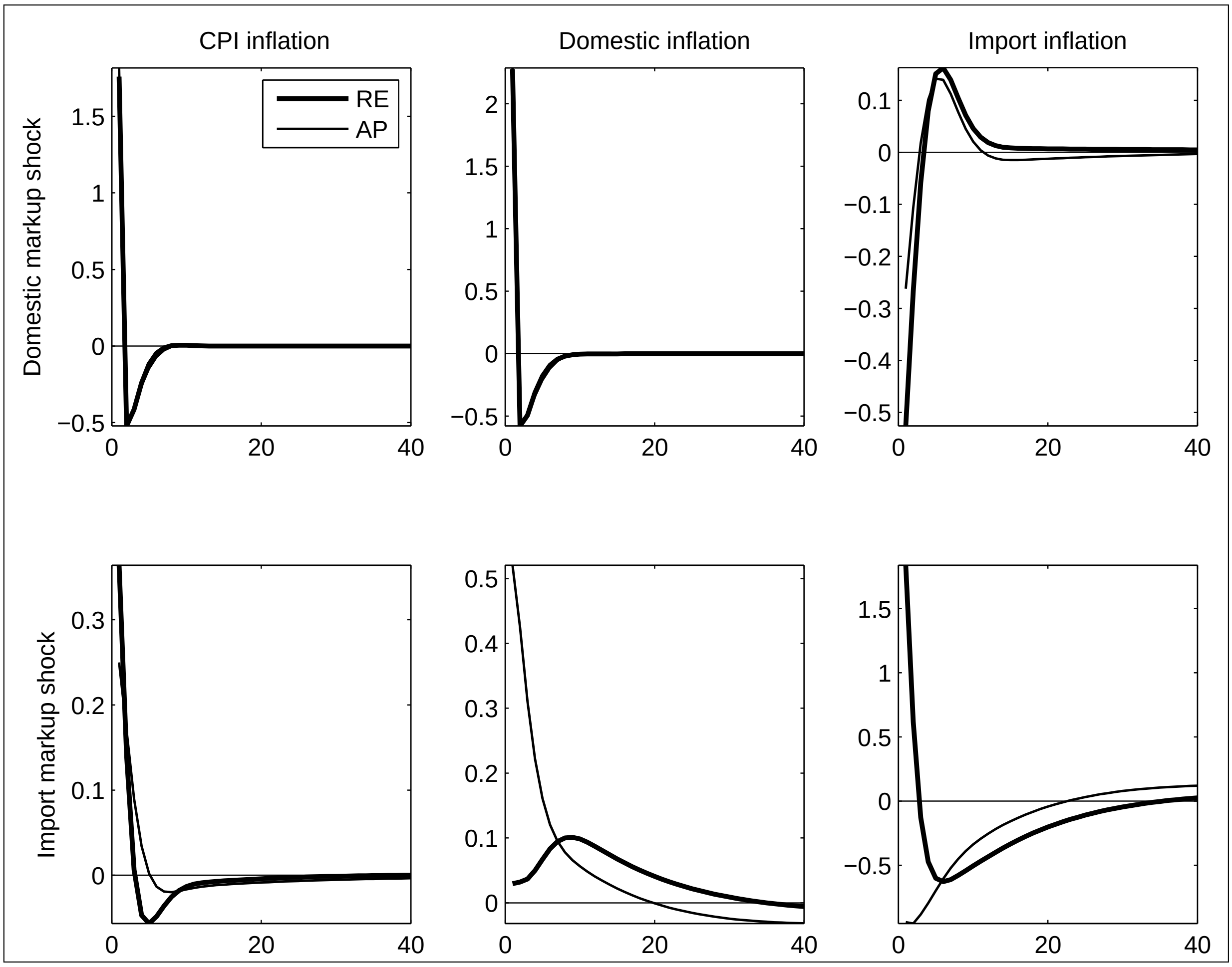
Note: The figure shows impulse responses of key variables to shocks (of one standard deviation) in the open-economy model with only domestic shocks when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The inflation rates are the annualized quarterly change in the respective price level. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 3: Impulse responses in open-economy model with only domestic shocks



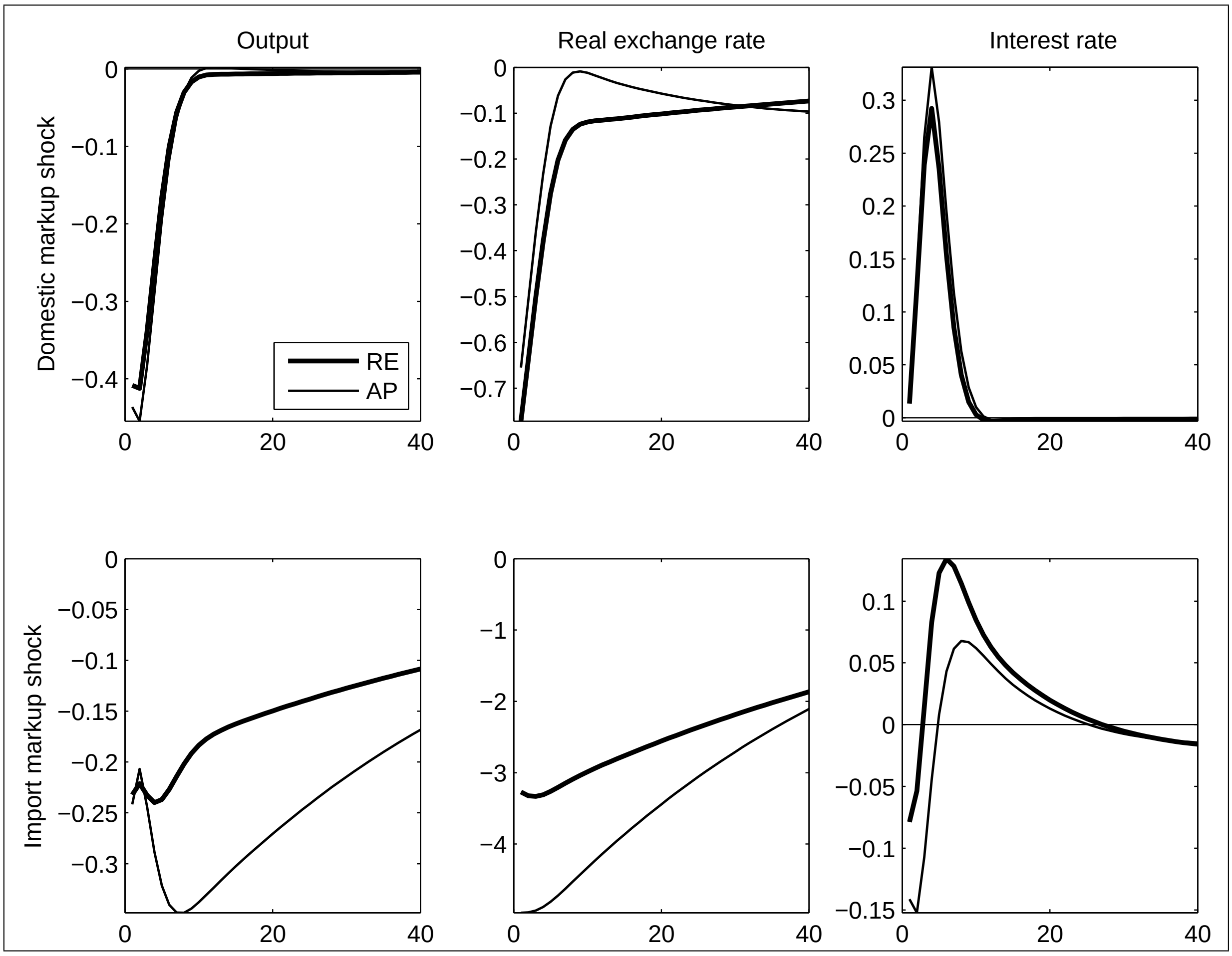
Note: The figure shows impulse responses of key variables to shocks (of one standard deviation) in the open-economy model with only domestic shocks when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The interest rate is expressed in annualized terms. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 4: Impulse responses to markup shocks in open-economy model with domestic shocks and shocks to the import price markup



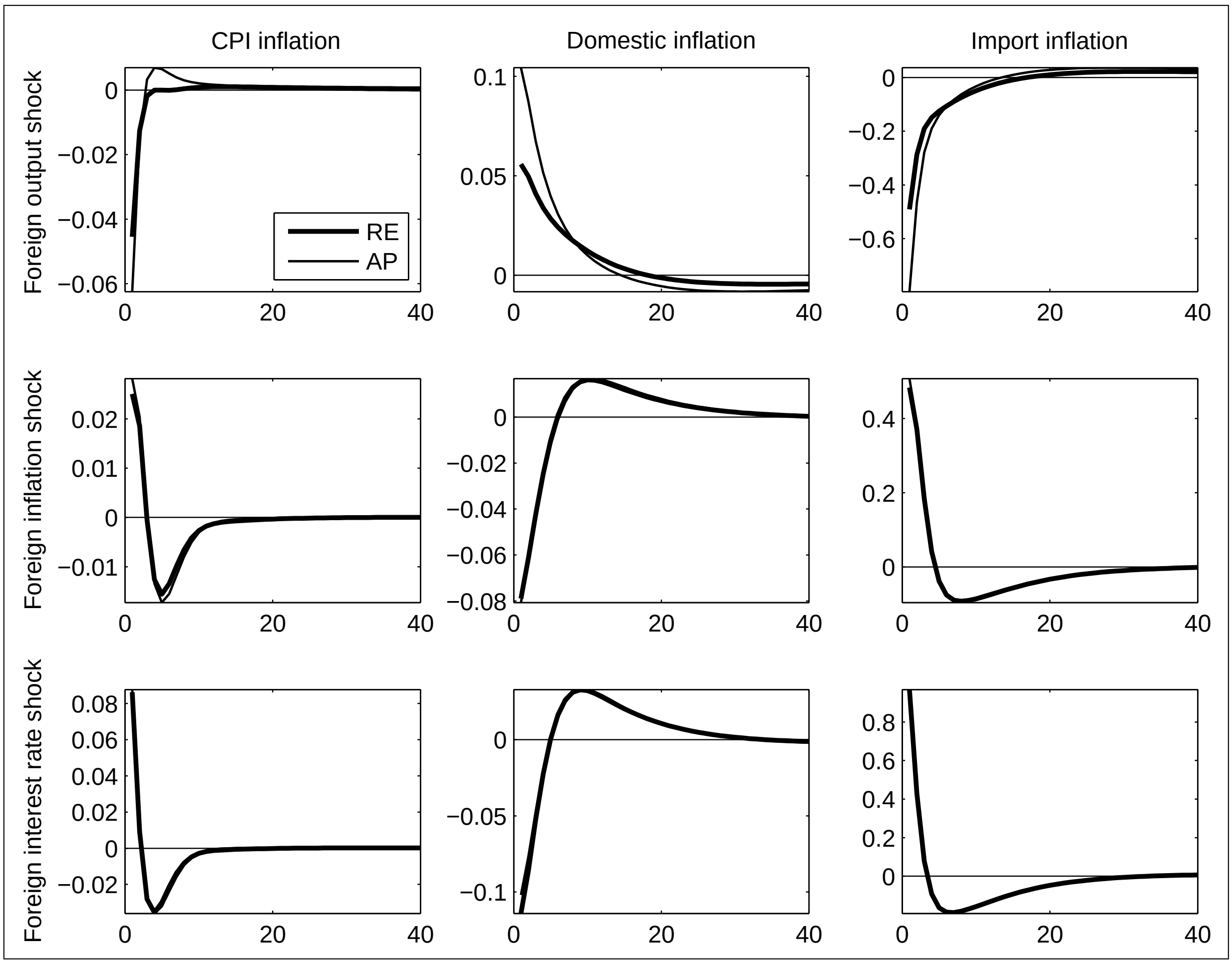
Note: The figure shows impulse responses of key variables to domestic and imported price markup shocks (of one standard deviation) in the open-economy model with domestic shocks and shocks to the import price markup when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The inflation rates are the annualized quarterly change in the respective price level. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 5: Impulse responses to markup shocks in open-economy model with domestic shocks and shocks to the import price markup



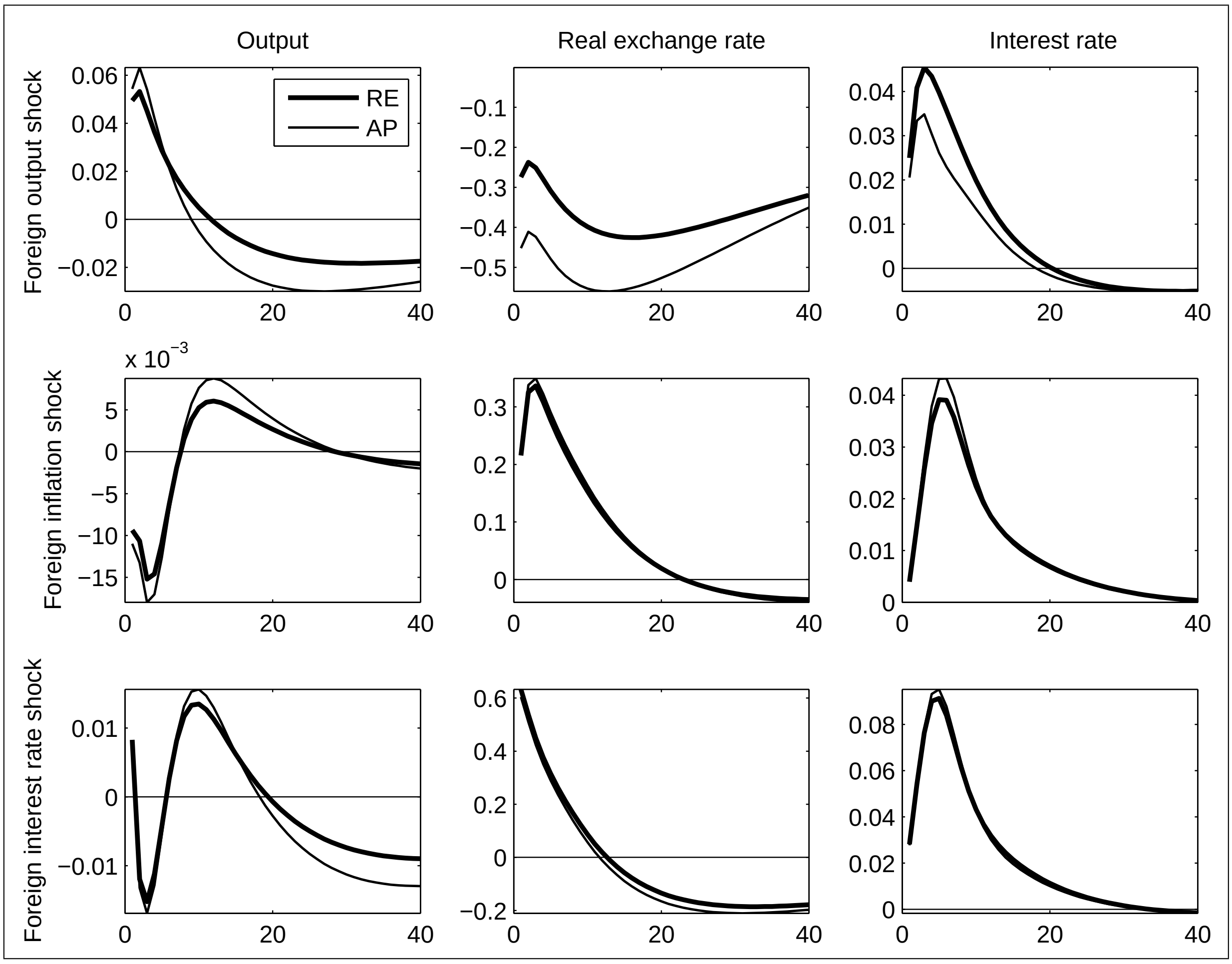
Note: The figure shows impulse responses of key variables to domestic and imported price markup shocks (of one standard deviation) in the open-economy model with domestic shocks and shocks to the import price markup when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The interest rate is expressed in annualized terms. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 6: Impulse responses to foreign shocks in open-economy model without exchange rate shocks



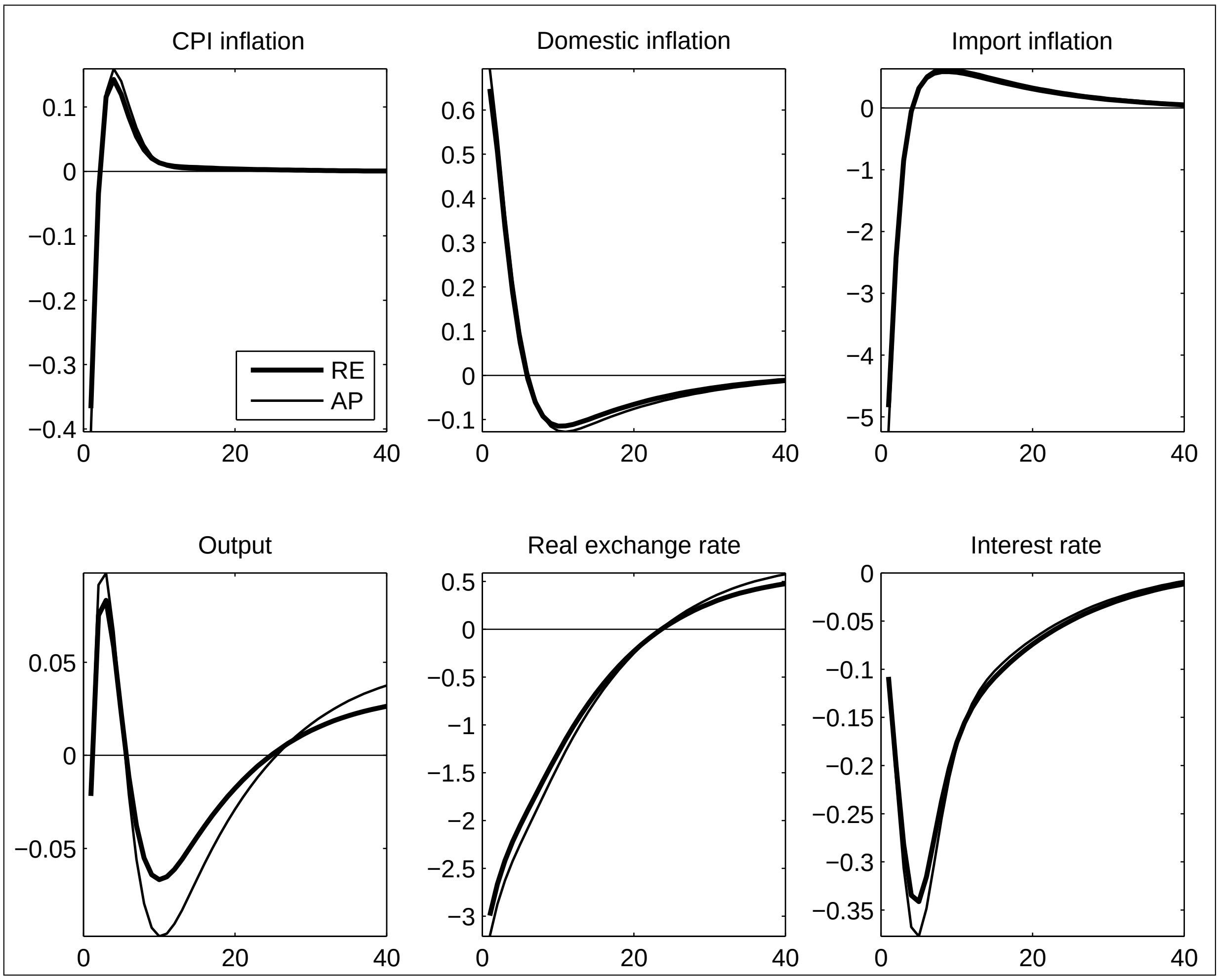
Note: The figure shows impulse responses of key variables to foreign shocks (of one standard deviation) in the open-economy model without exchange rate shocks when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The inflation rates are the annualized quarterly change in the respective price level. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 7: Impulse responses to foreign shocks in open-economy model without exchange rate shocks



Note: The figure shows impulse responses of key variables to foreign shocks (of one standard deviation) in the open-economy model without exchange rate shocks when monetary policy and specification errors are set with commitment. "RE" represents the outcome with rational expectations and non-robust monetary policy, "AP" is the outcome in the approximating equilibrium with robust policy. The interest rate is expressed in annualized terms. The preference for robustness is chosen to produce a detection error probability of 0.2.

Figure 8: Impulse responses to foreign exchange risk premium shock in open-economy model with all shocks



Note: The figure shows impulse responses of key variables to the foreign exchange risk premium shock (of one standard deviation) in the open-economy model with all shocks when monetary policy and specification errors are set with commitment. “RE” represents the outcome with rational expectations and non-robust monetary policy, “AP” is the outcome in the approximating equilibrium with robust policy. The inflation rates are the annualized quarterly change in the respective price level, the interest rate is expressed in annualized terms. The preference for robustness is chosen to produce a detection error probability of 0.2.