Simple Monetary Policymaking without the Output Gap*

Kai Leitemo*
Norwegian School of Management BI

and

Ingunn Lønning
SUNY College at Old Westbury

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Abstract

Several research contributions have argued that information about the output gap is essential for a good monetary policy rule. However, as pointed out by Orphanides (2001), there is considerable real-time uncertainty about the size of the output gap. The paper argues that simple monetary policy rules that rely exclusively on (survey-based) information about future and past inflation rates may be more efficient than optimized Taylor rules once real-time output gap uncertainty is accounted for. Even when only information about historical inflation rates is available, a very simple policy rule may be constructed that improves on the Taylor rule.

Keywords: Monetary policy, simple rules, uncertain output gap, Taylor rules.

JEL classification codes: E58, E52, and E47.

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* Corresponding author: Economics Department, Norwegian School of Management BI, PO Box 580, 1302 Sandvika, Norway. Tel: +47 67557477. E-mail: kai.leitemo@bi.no http://economics.no
1 Introduction

Recent literature on monetary policy typically finds that a monetary policy that responds to the output gap will help stabilize inflation and output. As Svensson (1997) shows, due to the effect of the output gap on inflation, an optimal inflation-targeting policy implies a monetary policy response to the output gap. In addition, the central bank may want to stabilize the output gap per se. Consequently, precise output gap data are of potentially great importance to the monetary policymaker.

However, the output gap is not readily observable. While the definition of the output gap is the deviation of output from some natural (or potential) level, the estimation of the output gap is complicated by controversies surrounding the definition\(^1\) and modelling of the natural level of output\(^2\) and by issues concerning the statistical measurement of output itself.\(^3\) A further complication is that policy is set in real time and not with the blessing of hindsight. Since output information arrives gradually, the real-time data on output are preliminary and associated with great uncertainty, leading to large revisions in output gap data over time, see for instance Orphanides (2003a) for documentation on US data. This real-time mismeasurement of the output gap represents a major problem for the implementation of policy strategies that rely on information about the current output gap.

In this paper, we suggest that the central bank use simple proxies for the output gap, either instead of, or in addition to the noisy real-time estimate of the output gap. We operate within a framework of simple instrument rules, i.e., rules that explicitly model the interest rate as a function of a limited set of specified state variables. An important aim of this and other contributions to the simple policy rule literature is to give policymakers a set of principles, insights or “rules-of-thumb” that can be used as guides in the conduct of policy. It can also be argued that simplicity, in addition to efficiency, is a desirable element if a commitment to a specific policy rule is to be credible.

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\(^1\) It should be noted that there seems to be a convergence towards the precise definition of the natural rate of output, advocated by Woodford (2003a), as the output level that would prevail with flexible prices and wages.\(^2\) See, e.g., Holden and Nymoen (2002) for a discussion of the problems associated with the method employed by the OECD in estimating potential output.
Our approach is to use the model of the economy to derive two proxies that in a simple way provide surprisingly much information about the unobservable output gap. Both output gap proxies are derived using one of the equations in the model, the Phillips curve. The first and simplest proxy (simple proxy) relies only on observation of current and past prices. The second proxy relies in addition on survey information about private sector price expectations (expectation-based proxy). Generally, it is possible to use the Kalman filter to derive an optimal estimate of an unobservable variable, here the output gap, based on the whole model. However, this will no longer be a "simple" rule to operate and explain for the central bank. Also, an important assumption behind our approach is that the private sector has full information regarding factors that influence own price setting so that surveys bring new information to the policymaker. As shown in Woodford and Svensson (2002) the procedure of using the optimal but complex Kalman-filter estimate of the output gap is made even more intricate as the separation principle breaks down in the case of asymmetric information between the private sector and the central bank, implying that the estimation of the output gap will depend on the specific policy rule in operation.

As a benchmark rule for comparison we choose the Taylor (1993) rule, which serves as the basic reference in the literature on simple policy rules. The rule states that the interest rate should respond to deviations in inflation from its target rate and to the output gap. Taylor (1993) shows that the rule prescribes a policy that seems to be in line with monetary policy followed in the Greenspan period, while Svensson (1997) finds that the Taylor rule is the optimal interest rate rule in a model that features a traditional backward-looking Phillips curve. The rule has been shown to perform well within various model frameworks (Taylor, 1999), also within the New Keynesian model tradition with extensive forward-looking behavior (Woodford, 2001). These features make the Taylor rule an attractive benchmark rule. In addition, we could have compared our results to the optimal commitment policy (see Svensson and Woodford, 2002, 2003) which would allow us to measure the loss of efficiency in using our simple approach to policy. However, since our main focus is on providing simple, yet good proxies for the output gap, such an approach would be interesting, but considered beyond the scope of this paper.

We show that by reacting to information in our proxies instead of or in addition to the noisy measure of the output gap, the central bank may improve the outcome of monetary policy. Although the improvement on the Taylor rule is noticeable even if responding only to the simple proxy, the use of the expectation-based proxy improves significantly on the Taylor rule. The result reached in the paper is robust to how strongly the policymaker weighs inflation stability versus output gap stability in the central bank loss function, and is also robust to inflation data being observed with a one quarter lag.

\footnote{For a discussion on measuring the real-time output gap, see Orphanides and Norden (2002).}
1.1 More on output gap uncertainty
A monetary policy rule based on real-time output gap data runs the risk of deviating strongly from the desired and intended policy due to output gap mismeasurement. Orphanides (2000, 2003a) studies the difference between the estimates of the output gap available in real time and the 1994 estimates for the output gap for the period 1980-1992 on US data. He finds the measurement errors to be large and persistent. Orphanides (2001) demonstrates how policy recommendations based on real-time data for the output gap differ considerably from those obtained with revised data. Thus, simple monetary policy rules based on the output gap may not be robust to output gap uncertainty.

1.2 Approaches to dealing with output gap uncertainty
Studies on how to deal with output gap uncertainty in simple policy rules have proceeded along two lines. The first one asks how output gap uncertainty influences the way the policymaker should respond to the output gap. The second asks what alternative indicator could substitute for the output gap in the policy rule.

Along the first line several studies show that the central bank in its policy should attach less weight to a variable the more uncertain it is, see, for instance, Smets (2002) and Rudebusch (2002b). This is the case if the observed uncertain variable enters the policy function directly, whether policy is conducted using simple rules or non-simple optimal targeting rules, which embody responses to all of the state variables that affect the target variable. On the other hand, if policy is based on an optimal targeting rule and uses an optimal Kalman-filter estimate of the uncertain variables, then certainty equivalence holds so that optimal policy should react to the uncertain variable as if it were observed with certainty.5

Along the second line of research, several studies (McCallum 1998) have suggested a policy rule that embodies a response to nominal income growth. The advantage of this approach is that such a rule does not require knowledge of potential output. Rudebusch (2002b) shows, however, that only for large output gap uncertainty and for particular model formulations, does nominal income targeting improve the performance of the Taylor rule. A simple nominal income growth rule also appears less robust to model uncertainty. Orphanides et al. (2000) suggest that the output gap in the Taylor rule should be replaced by growth in the output gap as the growth numbers undergo smaller revisions over time. This could potentially reduce the effect of an uncertain real-time estimate of the output gap. They find that such a rule may outperform the Taylor rule if output gap uncertainty is relatively high compared to the historical level of output gap uncertainty. The benefits from reducing the impact of

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4 The choice of proxies is also motivated by the simple fact that inflation observations often seem to play an important role as indicators for monetary policy in central bank statements.

5 See, Svensson and Woodford (2002, 2003), Orphanides (2003a) and Swanson (2004) for a discussion of the distinction between the two cases. See also Ehrmann and Smets (2003) for an application of the method to a model of the euro area.
output gap uncertainty must be weighed against the cost of the reduced stabilization effect obtained. An obvious shortcoming to using the output gap growth indicator is that this suggests a tightening of monetary policy if the economy grows, even if the output gap is negative and exerts a downward pressure on inflation. Orphanides and Williams (2002) suggest a rule where the change in the interest rate responds to the change in the output gap, while also responding to the uncertain level of the output gap. This will to some extent counter the shortcoming mentioned above.

1.3 Our approach

As noted above, our approach is to use information about past and future expected price movements and via the Phillips curve, construct proxies that extract information about the output gap.

We start out with the simplest proxy, which is the change in the actual rate of inflation. The motivation for this proxy follows from the inflation accelerationist argument: if output is above potential, inflation will gradually increase. Thus, the change in the rate of inflation will provide useful information about the output gap.

However, the change in the rate of inflation will also be affected by supply shocks and expectations about future inflation. Thus, we show that by using information about expected future and past inflation rates appropriately weighted, we can derive from the model a second expectation-based proxy for the output gap that is free from traditional mismeasurement errors, and linearly related to the true output gap up to a white-noise cost-push shock. The weights attached to past and expected future inflation rates in the proxy are directly related to the weights attached to these variables in the Phillips curve relationship. Under the assumption that prices are efficiently set based upon past information sets and therefore predetermined within each period, the proxy becomes linearly related to the private-sector expectation about the output gap and is not distorted by a cost-push shock. Since private sector pricing is driven by output gap expectations, the proxy reflects the variable driving inflation. If, in addition, inflation reacts to the output gap with some lag, the proxy will be linearly related to the true output gap with the same lag length.

Finally, we combine the two proxies in a third combination rule where we also include a reaction to the observed mismeasured output gap. Thus, we have a rule based on three variables that each provide some information about the true output gap.

The second part of the paper tests the performance of the three alternative rules and compares them to the performance of the Taylor rule under different assumptions about the amount and type of output gap uncertainty present in the data. We also include an analysis of the case where inflation is only observable with a lag. The relative performance of the proxy-based rules is studied within a New Keynesian model framework (see, e.g., Clarida, Gali and Gertler, 1999) as empirically specified in
Rudebusch (2002b). The rules close the model, and the effects of each rule on the variability of inflation and the output gap are derived using stochastic simulations.

Section 2 of the paper presents the model framework and derives the proxies. Section 3 evaluates the performance of the proxy rules and Section 4 concludes.

2. The model framework

Our choice of macroeconomic framework is that of the new Keynesian tradition with nominal rigidities. See Gali et al. (1999) and Woodford (2003a) for detailed presentations. The new Keynesian consensus on price dynamics can be expressed by a generalized Phillips curve of the form (see Woodford, 2003b, p. 76)\(^6\)

\[
\pi_t = E_{t-d} \left[ \mu \pi_{t+1} + (1 - \mu) \pi_{t-1} + \alpha_y y_t \right] + \epsilon_t, \tag{1}
\]

where \(\pi_t \equiv p_t - p_{t-1}\) is the rate of inflation, \(E_{t-d}\) is the rational expectations operator given information available at time \(t-d\), and \(\epsilon_t\) is a stochastic error term, referred to as a cost-push shock. \(y_t\) is the output gap, that is, the percentage deviation of output from the natural rate of output. Price setting is subject to an implementation lag of \(d \geq 0\) periods.\(^7\)

The pure New Keynesian Phillips curve derived from microfoundations asserts that \(\mu = 1\). This result can be derived within a model of staggered price setting with model-consistent expectations, as in Calvo (1983). It is then assumed that the output gap relates linearly to marginal costs, which affect price setting.\(^8\) Due to staggered price setting, price setters must act in a forward-looking manner, anticipating future marginal costs. Thus, this specification implies that inflation is not only determined by the present output gap, but also expected future output gaps. A forward-looking specification allows the inflation rate to jump when new information about marginal costs arrives that leads agents to revise their expectations.

With \(\mu = 0\), the specification collapses into the traditional accelerationist Phillips curve, and past inflation rates and hence past output gaps are determinants of inflation. From the point of view of optimizing behavior, it is difficult to find a rationale for backward-looking price setting. However, Roberts (1995, 1997) and Gali and Gertler (1999) argue that the existence of rule-of-thumb or non-rational price setters may introduce backward-looking terms in the Phillips curve. Fuhrer (1997) finds

\(^6\) We assume that the discount factor of the representative agent is unity.

\(^7\) A specification with \(d = 1\) is used, e.g., in Bernanke and Woodford (1997).

\(^8\) A reasonable interpretation is that the output gap is a representation of the squeeze on available resources, and indirectly the level of marginal costs faced by the price setters. There are other variables that will represent the level of marginal costs. For instance, Gordon (1997) finds that lagged values of the unemployment gap, the
that a value of $\mu_\pi = 0.2$ describes US inflation dynamics well but cannot rule out a value of zero. Based on various empirical studies Rudebusch (2002b) suggests a value of $\mu_\pi$ between 0 and 0.6. Estrella and Fuhrer (2001) discuss the dynamics implied by a purely forward-looking model and argue that such dynamics are unrealistic.

2.1 Aggregate demand

We consider a closed-economy model with only a single consumption good. The first-order condition for optimal consumer behavior implies a traditional smoothed consumption profile. Demand moves in the same direction as expected future demand, and depends, due to intertemporal substitution effects, on the real interest rate. The equilibrium real interest rate, $r^*$, is the interest rate consistent with product market equilibrium. The first-order condition in terms of the output gap may be written as

$$y_t = \mu_y E_t y_{t+1} + \left(1 - \mu_y \right) y_{t-1} - \beta_y (i_t - E_t \pi_{t+1} - r^*) + \eta_t,$$

where $y_t$ is the output gap, $E_t y_{t+1}$ is the expected output gap at time $t+1$ given the information available at time $t$, $i_t$ is the nominal interest rate, $E_t \pi_{t+1}$ is the expected inflation rate at time $t+1$ given the information available at time $t$, $r^*$ is the equilibrium real interest rate, and $\eta_t$ is a stochastic error term, representing demand shocks. The presence of a backward-looking term in the Euler equation, that is, $\mu_y < 1$, can be justified by for instance habit formation, where the utility of current consumption depends on previous consumption levels, see Fuhrer (2000) and McCallum (2001).

2.2 Deriving the simple output gap proxies

We now describe the various proxies and the monetary policy rules we study. As noted in the introduction, we shall only consider the class of simple monetary policy rules. We use the Taylor (1993) rule as a benchmark for evaluating the alternative rules. We start by exposing the problems with the Taylor rule under output gap uncertainty. Then we proceed to discuss our suggested proxy rules:

(i) A simple proxy rule where the output gap in the Taylor rule has been substituted by the actual change in the rate of inflation

(ii) An expectation-based proxy rule where the output gap in the Taylor rule has been substituted by a proxy based on changes in both expected and actual inflation rates

deviation of unemployment from NAIRU, i.e. the labor market equivalent of the output gap, are significant determinants of inflation.
(iii) A framework where the Taylor rule has been augmented by both of the proxies in (i) and (ii).

As noted above, and shown more explicitly below, we derive our proxies using the Phillips curve. Note, however, that the Euler equation will also carry potentially useful information about the actual output gap that, i.e., an efficient, but nonetheless complex, Kalman filter may utilize in order to achieve a better estimate. Note, however, that there is no simple way of extracting information from the Euler equation as it depends on lags of the dependent variable. The use of the Phillips curve, however, goes a long way in extracting information about the true output gap from the model, while maintaining the overall simplicity of the approach to monetary policymaking.

(o) The Taylor rule under output gap uncertainty

The Taylor rule is given by

$$i_t = r^* + \pi_t + \tilde{g}_T (\pi_t - \pi^*) + g_y y_t,$$

where $r^*$ and $\pi^*$ are the equilibrium real interest rate and the inflation target, respectively. $\pi_t$ is the four-quarter inflation rate. We normalize the equilibrium real interest rate and the inflation target to zero. Acknowledging that the policymaker needs to use the real-time estimate of the output gap, the Taylor rule can be expressed as

$$i_t = g_T^\pi \pi_t + g_y^\pi \hat{y}_t,$$

where $g_T^\pi \equiv \left(1 + \tilde{g}_T\right)$ and the real-time estimate of the output gap is given by

$$\hat{y}_t = y_t + n_t,$$

$y_t$ is the final estimate of the output gap, $\hat{y}_{t+i}$ is the period $t+i$ estimate of the output gap in period $t$, and $n_t$ is the measurement error. After inserting for $\hat{y}_t$ in the Taylor rule, we get

$$i_t = g_T^\pi \pi_t + g_y^\pi y_t + g_y^\pi n_t.$$

It is evident from equation (6) that the Taylor rule is sensitive to mismeasurement of the output gap, and that, depending on the value of $g_y^\pi$, the measurement error can have potentially large consequences for interest-rate setting.

(i) The simple proxy rule

We suggest first to avoid reacting to a direct but uncertain measure of the output gap by using the change in the rate of inflation as a proxy for the output gap. This is a strikingly simple proxy, derived from the inflation accelerationist proposition: whenever output exceeds its potential, inflation will
gradually increase and signal a positive output gap. The proxy will be contaminated by possible cost-push shocks and changes in expectations about future inflation that influence current price setting, and it will be an empirical issue whether this proxy provides sufficient information regarding the output gap to compete with the uncertain observation of the output gap.\footnote{Looking at the change in the rate of inflation as a proxy for the output gap is not a new idea, however. For example, as US unemployment fell in the last part of the 1990s, unemployment was first believed to have fallen below NAIRU, which indicates that there would be a positive output gap. However, when inflation did not}

By properly rearranging the Phillips curve of our theoretical model, the simple proxy is given by:

$$
\hat{y}_t = \pi_t - \pi_{t-1} = \alpha \varepsilon_{t-d} y_t + \mu \varepsilon_{t-d} \left( \Delta \pi_{t+1} + \Delta \pi_t \right) + \left( E_{t-d} \pi_{t+1} - \pi_{t-1} \right) + \varepsilon_t,
$$  \hspace{1cm} (7)

The simple proxy is proportional to the expected output gap up to a white-noise cost-push shock if $\mu = 0$ and $d \leq 1$, that is, if there is no forward-looking behavior in price setting and the implementation lag is no longer than one quarter.

The rule based upon the simple output gap proxy is consequently

$$
i_t = g_{\pi}^t \overline{\pi}_t + g_{\pi}^t \hat{y}_t = g_{\pi}^t \overline{\pi}_t + g_{\pi}^t \Delta \pi_t.
$$  \hspace{1cm} (8)

The simple proxy rule uses very little information and should therefore be easy to implement. However, it will also provide incomplete information about the output gap as variations in the proxy may reflect cost push shocks or movements in inflation expectations. Thus, we may be able to create a better proxy by including information regarding inflation expectations if the central bank can easily obtain surveys of private sector inflation expectations.

\textbf{(ii) The expectation-based proxy rule}

By taking expectations as of time $t-d$, the Phillips curve (1) may be rearranged to

$$
(1 - \mu) E_{t-d} \left( \pi_t - \pi_{t-1} \right) - \mu E_{t-d} \left( \pi_{t+1} - \pi_t \right) = \alpha \varepsilon_{t-d} y_t + E_{t-d} \varepsilon_t,
$$  \hspace{1cm} (9)

which shows that a proxy $\hat{y}_t = (1 - \mu) E_{t-d} \left( \pi_t - \pi_{t-1} \right) - \mu E_{t-d} \left( \pi_{t+1} - \pi_t \right)$ is proportional to the expected output gap up to a white-noise cost-push shock. However, for $d \geq 1$, we observe that the proxy becomes proportional to the expected output gap without being affected by the cost-push shock. Thus, with access to private sector inflation expectations, the central bank may, based on the Phillips curve, create a proxy that is closely related to the true output gap.
We formulate a second alternative policy rule that uses the expectation-based proxy for the output gap as a replacement for the central bank estimate of the output gap, i.e.

\[ i_t = g^e_x \pi_t + g^e_p \hat{y}_t. \]  

(10)

By using the definition of \( \hat{y}_t \), the rule may be written as

\[ i_t = g^e_x \pi_t + g^e_p \left( \alpha_y E_{t-d} y_t + E_{t-d} e_t \right), \]

for \( d \geq 1 \).

(11)

In general, there is a response to both the expected (true) output gap and the cost-push shock. If \( d \geq 1 \), there is only a response to the expected output gap, which drives inflation in the model. Hence, the proxy avoids output gap mismeasurement altogether. 10

(iii) The augmented rule

In the last alternative rule, which we denote the augmented rule, we combine both proxies in one rule and keep the real-time measure of the output gap:

\[ i_t = g^a_x \pi_t + g^a_p \hat{y}_t + g^a_y \hat{y}_t + g^a_{yp} y_p. \]  

(12)

2.3 The empirical specification

For the simulations we use the estimation of the model in Rudebusch (2002b). Rudebusch argues that the theoretical model is typically based on annual leads and lags. At a quarterly frequency, this translates into longer leads and lags in the empirical model. Further, Gali and Gertler (1999), Christiano et al. (2001) and Amato and Laubach (2003) show within the Calvo (1983) framework that lags in an otherwise forward-looking Phillips curve can be rationalized by a fraction of the price setters being subject to indexation of prices or simple rules-of-thumb pricing behavior. Rudebusch specifies the Phillips curve as

\[ \pi_t = \mu_x E_{t-1} \pi_{t+3} + (1 - \mu_x) \left( \alpha_{x1} \pi_{t-1} + \alpha_{x2} \pi_{t-2} + \alpha_{x3} \pi_{t-3} + \alpha_{x4} \pi_{t-4} \right) + \alpha_y y_{t-1} + \epsilon_t, \]

(13)

increase, the lower unemployment rate was interpreted as a fall in the NAIRU. Thus, the lack of increase in the inflation rate was interpreted as a sign that the output gap had not increased.

10 In the case with no implementation lags (\( d=0 \)), the expectation-based proxy will on average be a better proxy for the output gap if the variance of its mismeasurement error is smaller than for the mismeasured output gap, which is true for \( \left( \frac{1}{d} \right)^2 \sigma^2_e < \sigma^2_y \).
where $\bar{\pi}_{t+3} = \sum_{j=0}^{3} \pi_{t+3-j}$ is the annual inflation over the coming year. Rudebusch estimates this Phillips curve on US data and finds coefficient values of $\mu_\pi = .29$, $\alpha_{s_1} = .67$, $\alpha_{s_2} = -.14$, $\alpha_{s_3} = .40$, $\alpha_{s_4} = 1 - \alpha_{s_1} - \alpha_{s_2} - \alpha_{s_3} = -.07$ and $\alpha_y = .13$. The standard error is $\sigma_\pi = 1.012$.

For the empirical aggregate demand equation, Rudebusch (2002b, Appendix A) suggests:

$$y_t = \mu_\pi E_{t-1} y_{t+1} + (1 - \mu_\pi) \left( \mu_2 y_{t-1} + \mu_3 y_{t-2} \right) - \beta_\pi (i_{t-1} - E_{t-1} \bar{\pi}_{t+3} - r^*) + \eta_t. \quad (14)$$

Fuhrer (2000) shows how lags in the output gap in the demand function follow from utility maximization under habit formation. Based on a combination of his own estimates and figures appearing in other works (Fuhrer, 2000), Rudebusch suggests plausible coefficient values for the US economy, with $\mu_\pi = 1.15$, $\mu_\zeta = -0.27$, $\beta_\pi = 0.09$ and $\mu_\zeta$ approximately equal to 0.3. The standard error is estimated at $\sigma_\pi = .833$. The output gap in the Phillips curve and the interest rate in the demand equation are both lagged one period. The information set is also lagged one period and output and inflation are both regarded as predetermined variables. This captures the empirically sluggish effect of monetary policy. Acknowledging uncertainty associated with the coefficient estimates, we perform some robustness checks of the monetary policy rules to changes in the various coefficient values.

Based on the empirical Phillips curve (13) with an implementation lag of one period, the simple output gap proxy is given by

$$\hat{y}_t^{\pi} \equiv \pi_t - \bar{\pi}_{t-1} = \mu_\pi E_{t-1} (\bar{\pi}_{t+3} - \bar{\pi}_{t-1}) + \alpha_y y_{t-1} + \epsilon_t \quad (15)$$

where $\bar{\pi}_{t-1} = \alpha_{\pi_1} \pi_{t-1} + \alpha_{\pi_2} \pi_{t-2} + \alpha_{\pi_3} \pi_{t-3} + \alpha_{\pi_4} \pi_{t-4}$. If $\mu_\pi = 0$ the proxy is a linear function of the true output gap and the cost-push shock. For $\mu_\pi > 0$, however, the proxy will also be a function of expected future and lagged changes in the rate of inflation.

The expectations-based output gap proxy may be constructed along the lines described in Section 2.3 as

$$\hat{y}_t^\pi = (1 - \mu_\pi) E_{t-1} (\pi_t - \bar{\pi}_{t-1}) - \mu_\pi E_{t-1} (\bar{\pi}_{t+3} - \bar{\pi}_t) = \alpha_y y_{t-1}. \quad (16)$$

This proxy will thus be linearly related to the one-period lagged true output gap, which is a state variable in the model, affecting prices.

In the remainder of the paper, we shall evaluate the performance of the proxy-based rules in (8) and (10), with the empirical proxies given in (15) and (16). The performance of the rules will be compared to the Taylor rule in (4).
2.4 Assumptions about the mismeasurement of the output gap

We need to make assumptions about the form and size of the measurement error in the output gap. The real-time estimate of the output gap at time $t$ is given by (5). Orphanides et al. (2000) suggest that the measurement error of the output gap can be approximated by a first-order autoregressive process,

$$n_t = \rho_n n_{t-1} + \xi_t.$$  \hfill (17)

This stochastic process captures the potential persistence in the measurement error: some part of the mismeasurement of the output gap in the present quarter is expected to be carried over into the next quarter’s estimate. Orphanides et al. (2000) provide estimates of the parameters in (17) for three different periods covering 1966-1994. Their estimates are reported in Table 1.

<table>
<thead>
<tr>
<th>Time period</th>
<th>$\hat{\rho}_n$</th>
<th>Sd ((\xi)) (in per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline case – output gap revisions 1980:Q1 – 1994:Q4</td>
<td>0.84</td>
<td>0.97</td>
</tr>
<tr>
<td>Worst case (High uncertainty) – output gap revisions 1966:Q2 – 1994:Q4</td>
<td>0.96</td>
<td>1.09</td>
</tr>
<tr>
<td>Best case (Low uncertainty) – capacity utilization revisions 1980:Q1 – 1994:Q4(^{11})</td>
<td>0.80</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Note: We adopt the baseline case figures as our baseline and will refer to the two other cases either as levels of “low” and “high” uncertainty or as best or worst-case levels of uncertainty.

3 Analysis of the rules

We now turn to the analysis of how useful either of the proxies is relative to an imperfect measure of the output gap. We do this by comparing the loss of the central bank under the different policy rules, with varying assumptions about the degree of output gap mismeasurement. Section 3.1 discusses the central bank’s objectives. Section 3.2 evaluates the relative usefulness of the indicators included in the rules under varying configurations of output gap uncertainty. Section 3.3 and 3.4 discuss the efficiency of the rules under different assumptions about the relative weight on output gap versus inflation stabilization in the central bank loss function. In section 3.4 we study how a one quarter lag in information about the inflation rate will affect our conclusions.

3.1 Central bank preferences

We assume that the central bank has a conventional quadratic loss function with periodic loss given by

$$L_t = (1 - \lambda) \left( \bar{\pi}_t - \pi^* \right)^2 + \lambda y_t^2 + \nu \left( i_t - i_{t-1} \right)^2.$$  \hfill (18)

As is shown in Rudebusch and Svensson (1999), we have that

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\(^{11}\) Orphanides et al. find that revisions in measures of capacity utilization in manufacturing have been smaller over the actual period and use these figures to give a possible measure of a future “best case” output gap uncertainty.
\[ \lim_{\beta \to 1} (1 - \beta) E_0 \sum_{t=0}^{\infty} \beta^t L_t = EL_t, \]  
which implies that if \( \beta = 1 \), the intertemporal loss function can be written as the weighted sum of the unconditional variances of the goal variables, i.e.,

\[ EL_t = (1 - \lambda) \text{var}(\pi_t) + \lambda \text{var}(y_t) + \nu \text{var}(i_t - i_{t-1}). \]  

We assume that the central bank minimizes equation (20) subject to the Taylor rule or either of the proxy-based rules, and the model, while taking into account that the estimate of the output gap is imperfect. As baseline values of the parameters in the loss function, we set \( \lambda = .5 \) and \( \nu = .01 \). A value of \( \lambda = .5 \), with equal weight on inflation and output gap variability, is in line with the assumption made in Orphanides and Williams (2002), Smets (2003) and others. Calvo-pricing with reasonable parameters typically suggests that the central bank within a welfare-optimizing framework should care relatively more about inflation variability (see Woodford, 2003a). One reason for attaching more weight to output gap fluctuations could be that these may be perceived as more costly than indicated by the standard representative agent theory, e.g. due to possible labor market imperfections and loss of labor skills during periods of low activity. However, in Section 3.3, we show that the relative performance of the rules is independent of the relative weight on inflation. We also include a term for interest rate change variability, as done, e.g., by Rudebusch (2002b), Orphanides and Williams (2002) and others. One reason for this inclusion may be that large interest rate changes may cause financial instability (Cuikerman, 1990). Another reason from our point of view is that monetary policy rules with large coefficients may be in danger of prescribing a violation of the zero-bound on nominal interest rates. For a given variability of inflation and output (and other indicator variables), a low coefficient on these variables reduces the likelihood for such a prescription. A concern for interest-rate smoothing may be a proxy for the policymaker’s wish to avoid this. Other approaches to dealing with the zero-bound is to make the central bank concerned about the variance in the level of the interest rate (Woodford 2003a) or even to derive optimal policy subject to the zero-bound constraint (Adam and Billi, 2004a, 2004b). We attach only a small parameter to the interest rate smoothing term in the loss function which nevertheless allows the optimal coefficients in the rules to have reasonable magnitudes.

### 3.2 The usefulness of the proxies
Figure 1 plots the optimized coefficients in a Taylor rule for different levels of output gap uncertainty, represented both by different levels of persistence and shock variability in the measurement error process. A number of interesting observations can be made.

\[ \text{The expected loss for using the simple policy rules is computed using an algorithm described in Söderlind (1999). We used the optimization package in Gauss in order to search for the coefficients that minimize the loss.} \]
First, as also found elsewhere (see Smets (2002) and Rudebusch (2002b)), the optimal coefficient on the output gap in the Taylor rule decreases as the actual level of output gap uncertainty increases. The intuition for this result is straightforward: as the reliability of an indicator is reduced, one should place less emphasis on the information it conveys. Second, this result is independent of whether the increased uncertainty comes in the form of higher persistence or higher shock variability.

Third, the no-uncertainty coefficient levels are much larger than coefficients obtained from empirical estimation of Taylor rules on the US economy (see, e.g. Taylor (1993, 1999) and Judd and Rudebusch (1998)). However, as output gap uncertainty increases towards a more realistic level, the optimized coefficients get closer to their associated empirical counterparts, suggesting that taking into account output gap uncertainty is an important element of actual policymaking.

Fourth, the coefficient on inflation also falls when the degree of uncertainty increases, indicating that higher output gap uncertainty should moderate not only the policy reaction to the output gap, but also the reaction to the inflation rate. This result is in line with similar studies on the effect of output gap uncertainty on simple monetary policy rules, see for instance Smets (2002) and Rudebusch (2001), but contrasts with the recent literature on the consequences of output gap uncertainty in an optimal targeting rule. Swanson (2004) shows that certainty equivalence holds for all coefficients in the
policy reaction function when policy reacts to the optimal forecasts of the unobserved or mismeasured variables. However, in making an optimal estimate on the uncertain variable, the weight on the uncertain variable(s) should decrease, while the weight on variables measured with greater accuracy should increase, as their relative informational value increases.

Swanson (2004) suggests that the discrepancy between the results may be explained by the fact that simple policy rules constrain the central bank to react to a limited number of state variables, where each variable may be related to other state variables to which monetary policy may want to adjust its reaction. This may lead to an ambiguous effect on the policy response to the inflation rate when output gap uncertainty increases.

Turning to our specific model, a change in the inflation rate may signal either a change in the output gap or a cost-push shock. In the case of a demand shock a stronger policy reaction to the inflation rate can substitute for a reaction to an increasingly imprecisely measured output gap. In the presence of cost-push shocks, however, a stronger reaction to the change in the inflation rate will destabilize output. As the noise in the output gap measure increases, the coefficient on the output gap falls, and the central bank’s capability of stabilizing the output gap in the presence of cost-push shocks is reduced. The optimal response is then to reduce the coefficient on inflation in order to react less to inflation driven by cost-push shocks and thus to alleviate the problems associated with output gap stabilization. Figure 1 confirms this. If, however, as an experiment, we remove the impact of the cost-push shocks, the results are turned around. Figure A0 in the Appendix plots the reaction coefficient on both inflation and the output gap in this situation. As expected, when output gap uncertainty increases, there is a tendency for the optimal coefficient on the inflation rate to increase. As the persistence in the output gap error increases, there is a uniform increase in the coefficient on the inflation rate. As the noise in the output gap increases, the coefficient on the inflation rate increases at first, but if the uncertainty becomes extremely high, the coefficient on the inflation rate falls towards a limit as the coefficient on the output gap goes to zero. We ascribe the latter result to lags in the empirical model, which imply that a reaction to the change in the inflation rate is a reaction to the lagged output gap. Thus, a very large reaction to the lagged output gap may have a destabilizing effect, especially because the interest rate works with a lag on output.

Now turning to the other rules, the optimized coefficients in the purely proxy-based rules will of course be independent of the level of output gap uncertainty, as the rules do not use the output gap as an indicator. The coefficients in the augmented rule, which includes a response to both proxies in addition to the mismeasured output gap, will depend on output gap uncertainty. We can study the relative value of each of the indicators by considering their associated coefficients in this rule.

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13 As the standard deviation of the measurement error shock varies, the persistence is kept at its baseline value. As the persistence is varied, the standard deviation of the measurement error shock is kept at its baseline value.
Figure 2. The optimized coefficients in the augmented proxy rule and the relative loss compared to the optimized Taylor rule.

Note: The upper charts show how the coefficients in the encompassing rule change as the standard deviation of the measurement error shock (left) and the persistence in the measurement error process increase. The lower charts show the associated welfare gain over the optimized Taylor rule. Vertical dashed lines show the benchmark assumptions about the measurement error process.

The upper parts of Figure 2 show the optimized coefficients for the augmented rule for different forms and size of the output gap mismeasurement and the lower parts show the associated welfare gain over the optimized Taylor rule.

The coefficient on the output gap declines as output gap uncertainty increases, while the coefficients on each of the proxies increase. The optimal coefficients on both of the proxies are relatively low when there is no output gap uncertainty.\(^{14}\) This implies that the augmented rule is close to a Taylor rule. As output gap uncertainty increases, both of the proxies’ efficiency improves relative to that of the real-time estimate of the output gap. In the limit, the coefficient on the output gap is zero. For baseline output gap uncertainty, the optimal augmented rule attaches some weight to all indicators.

\(^{14}\) The optimized coefficient on the expectation-based proxy seems very high at first. However, using the expression for the expectation-based proxy with \(\alpha_y = .13\) we get that for a coefficient value of 13, as is the baseline value, a one percentage point increase in the output gap should lead to a 1.69 percentage point increase in the interest rate.
of inflationary pressure, although the coefficient on the output gap is relatively small compared with the no-uncertainty case.

At benchmark levels of output gap uncertainty, the inclusion of the proxy terms implies a welfare gain of 30 per cent over the optimized Taylor rule. Regarding the proxies’ relative importance, simulation results (not shown) imply that there are only marginal additional benefits to including the simple proxy in the rule given that the expectation-based proxy is included. If the expectation-based proxy were to be excluded, however, the rule would perform only about 10 per cent better than the optimized Taylor rule under benchmark uncertainty, a significant drop in performance.

3.3 Welfare loss and relative efficiency of the policy rules
We now turn to consider the welfare implications by comparing the standard deviations of the goal variables and the resulting losses from using the different rules. First, we consider the efficiency frontiers of the rules. These frontiers trace out the minimum standard deviation of the goal variables as the relative weight on the output gap \( \lambda \) increases from 0.1 to 0.9 in the period loss function (18). Due to the relatively large number of rules, we have plotted the efficiency frontier in several figures.

**Figure 3a.** Efficiency frontiers for the Taylor rule (short-dashed lines) and the proxy-based rules under different levels of output gap uncertainty.

Note: The efficiency frontiers show the standard deviation of inflation and output resulting from the optimized rules as \( \lambda \) increase from 0.1 to 0.9, different degrees of output gap uncertainty considered. Note that the proxy-based rules are independent of output gap uncertainty.
Figure 3a shows the efficiency frontier for the proxy-based rules as long-dashed lines, whereas the efficiency frontiers for the Taylor rule at different levels of output gap uncertainty are shown as short-dashed lines. Figure 3b shows the efficiency frontier for the augmented rule, and also the Taylor rule for reason of comparison.

For baseline level of output gap uncertainty, the results suggest that both of the proxy rules are more efficient than the optimized Taylor rule. Interestingly, the result is robust to the choice of $\lambda$ as the efficiency frontiers for the proxy rules do not cross but is always below the efficiency frontier of the Taylor rule. The important benefits from using the expectation-based proxy is also evident in the figure as the expectation-based rule delivers substantially lower variability in the goal variables compared to the simple proxy rule for all choices of $\lambda$. It even matches the performance of the Taylor rule at low output gap uncertainty.

**Figure 3b.** Efficiency frontiers for the Taylor rule (short-dashed lines) and the augmented rule under different degrees of output gap uncertainty.

Note: The two short-dashed lines denote the efficiency frontier for the Taylor rule under baseline and no uncertainty. The long-dashed lines denote the efficiency frontier for the augmented rule for high, baseline, low and no uncertainty. The lines for the two rules under no uncertainty almost fully overlap. See also note for Figure 3a.

The performance of the augmented rule should at least match the performance of the expectation-based rule, since it includes a response to the same proxy but also to the simple proxy and, more importantly,
a response to a mismeasured output gap. The efficiency frontier for the augmented rule will therefore always be below that of the expectation-based rule, especially if output gap uncertainty is low. Moreover, the augmented rule will depend on the degree of output gap uncertainty assumed.

Figure 3c shows that the performance of the augmented rule is substantially better than that of the Taylor rule at baseline output gap uncertainty. Moreover, the efficiency frontiers for the augmented rule under baseline and high output gap uncertainty almost overlap, suggesting that the usefulness of the central bank output gap estimate is rather low for both uncertainty scenarios given that the central bank has access to both output gap proxies.

In an expanded working paper version of this paper we discuss the sensitivity of the conclusions reached to changes in model coefficients and increased weight on interest rate changes in the loss function. Whereas moderate changes in the model coefficients can either make the simple proxy rule improve and worsen relative to the Taylor rule, both the rule using the expectation-based proxy and the augmented rule rank better than the Taylor rule, even for relatively large changes in the coefficients. The performance of the proxy-based rules deteriorates relatively quickly, however, if the weight on interest rate movements increases. The reason is that the optimized proxy rules respond strongly to a cost-push shock since this shock affects all arguments in the proxy rules in the same direction.

3.4 Inflation observed with a lag
In the previous sections we assumed that inflation is immediately observed with no lags. Although it can be argued that inflation information is more readily available and observed with less imprecision (Orphanides, 2001) than output gap information, the assumption may seem to favor our suggested proxies and rules. In this section we therefore study how the relative performance of the rules is affected assuming that inflation is observed with a one quarter lag. In all rules, this implies that the current inflation term is replaced with lagged inflation. The simple proxy is also lagged by one period, now being the change of inflation in the previous quarter. Note, however, that the expectation-based proxy does not rely on the observation of current inflation, but rather on the previous quarter’s expectation of this period’s inflation rate, and is therefore unaffected by the information lag. Figure 3c shows the efficiency frontier for all four rules under baseline uncertainty. For comparison we include the frontiers based both on lagged and current inflation information. The lower diagram in the figure shows the associated loss for each rule caused by the loss of inflation information, for different specifications of $\lambda$. 

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The most striking result is the loss of efficiency for both the Taylor rule and the simple proxy rule. Information about current inflation is evidently important for both rules. For the Taylor rule, the lack of current inflation information is relatively more severe if the central bank is mostly concerned about inflation stabilization. If the central bank is more concerned with output gap stabilization, the reduction in inflation information is less severe as the output gap estimate is the important indicator variable and is unaffected. In the case of the simple proxy rule, the increase in loss is more independent of the weight on output in the loss function as both indicators in the rule are affected by the loss of inflation information. The expectation-based and the augmented rules are much less sensitive to the loss of information, but as for the Taylor rule, more so if the central bank attaches a large weight on inflation stabilization.

4 Concluding remarks

Literature on monetary policy asserts that a central bank that aspires to stabilize inflation around some target level benefits greatly from responding to good estimates of the output gap. Empirical work shows, however, that despite the resources available at the Federal Reserve, the real-time estimate of
the US output gap shows large and persistent measurement errors. Thus, interest-rate responses to the output gap may involve large and persistent policy errors. Given that most central banks do not possess the same resources as the Federal Reserve, we do not expect the US estimates to be any worse than others.

We show that it is possible to use the model to derive proxies for the output gap that are based on observable pricing information. Although it is not optimal to disregard information about the output gap completely, we start by formulating alternative policy rules to the Taylor rule that depend on information only about price movements. We show that such an approach may be superior to using optimized Taylor rules that do require responses to an output gap indicator. Also, including the noisy output gap information in the policy rule improves only slightly on performance. More specifically, our main conclusions can be summarized in three points.

First, a simple proxy based on changes in the actual inflation rate is slightly more useful than the real-time estimate of the output gap under baseline assumptions about output gap uncertainty. The result hinges, however, on the degree to which the policymaker prefers interest-rate smoothing, as the simple proxy rule implies a more active policy. Second, a proxy based on inflation expectations rather than the actual inflation rate or the real-time output gap may substantially improve policy. This result is independent of the degree to which the central bank prefers inflation to output gap stability.

Third, an augmented rule that includes the observation of the real-time output gap in addition to the two proxies represents only a small improvement in the performance compared to the expectations-based rule when the policymaker has full information about the degree of uncertainty.

Since the simple proxy rule responds to the actual increase in the rate of inflation rather than the expected determinants of future inflation, it may be more robust to other models, which assign a different role to the output gap in affecting future inflation. Even better, the expectation-based proxy rule that uses survey-based information about private-sector inflationary expectations leads to a contractionary policy if the price setters expect inflation to increase. The proxy-based rules may therefore be more robust to model uncertainty than rules that rely directly on output gap information.

Moreover, our output gap uncertainty figures are based on an assumption that the latest estimates of the output gap are the “true values.” The uncertainty tied to the final figures may imply that the true uncertainty in the output gap is larger than what we have used in our simulation. Finally, it may be possible to filter out some of the effects of cost-push shocks on prices using policy judgment. All these aspects may work to increase the relative desirability of the proxy-based rules in practical policymaking.
References


Woodford, Michael (2003c). ‘Inflation Stabilization and Welfare,’ Ch. 6 in Woodford (2003a)