Model Uncertainty and Delegation: A Case for Friedman's k-percent Money Growth Rule?*

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Abstract

Model uncertainty affects the monetary policy delegation problem. If there is uncertainty with regards to the determination of the delegated objective variables, the central bank will want robustness against potential model misspecifications. We show that with plausible degree of model uncertainty, delegation of the Friedman rule of increasing the money stock by k percent to the central bank will outperform commitment to the social loss function (flexible inflation targeting). The reason is that the price paid for robustness under flexible inflation targeting outweighs the inefficiency of money growth targeting. Imperfect control of money growth does not change this conclusion.

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1. Introduction

During the last 15 years in many countries, monetary policy has been formulated with an objective to stabilize inflation directly, a regime known as *inflation targeting*. Thus, many central banks, with perhaps the prominent exception of the European Central Bank, have abandoned regimes of targeting variables such as money growth or the exchange rate. Accordingly, inflation-targeting central banks put much resources into understanding the link between the instruments of monetary policy and inflation in order to establish what information is relevant and how to use this information most efficiently. Regardless of the effort put into describing the monetary transmission mechanism, however, it is unlikely that central banks ever achieve a complete understanding of this mechanism and thereby eliminate all model uncertainty. Incomplete understanding creates a need for monetary policy design to work well under alternative descriptions (models) of the economy and thus be robust to model uncertainty.

Guarding against model uncertainty has been a central topic in monetary policy for a long time, dating at least back to Friedman (1960) and his book *A Program for Monetary Stability*. In his famous book, Friedman discusses the choice of monetary policy target. He advocates money growth targeting and specifically warns against adopting price stability as an operational objective for monetary policy due to the high degree of model ("link") uncertainty between monetary policy and prices:

> [...] the link between price changes and monetary changes over short periods is too loose and too imperfectly known to make price level stability an objective and reasonably unambiguous guide to policy. [...] [T]here is much evidence that monetary changes have their effect only after a considerable lag and over a long period [...] (p. 87)

Friedman doubts the usefulness of state-contingent rules in monetary policymaking due to the uncertainty related to magnitude and lags in which the economy reacts to the policy stimulus.

> Under these circumstances, the price level - or for that matter any other set of economic indicators - could be an effective guide only if it were possible

to predict, first, the effects of non-monetary factors on the price level for a considerable period of time in the future, second, the length of time it will take in each particular instance for monetary policy actions to have their effect, and third, the amount of effect of alternative monetary actions. (p. 88)

For these reasons, Friedman suggests using policy rules that are invariant to detailed knowledge about the dynamics of the economy. Accordingly, he advocates the well-known constant k-% money growth rule:

> [...] The stock of money [should be] increased at a fixed rate year-in and year-out without any variation in the rate of increase to meet cyclical needs. (p. 90)

It is widely agreed that money growth targeting will stabilize inflation around a constant rate provided there are no change in the steady-state output growth rate and no change in the steady-state velocity growth rate. However, the rule will typically deviate from the optimal policy derived from minimizing the impact of price inertia on economic welfare in a setting with complete understanding of the economy.

In this paper we show that the degree of model uncertainty has implications for the choice of the loss function (mandate) for the central bank. Model uncertainty may indeed be a reason for the central bank should be delegated a loss function that is different from the social loss function. The argument is as follows: If there is high model uncertainty regarding the variables in the welfare function (typically inflation and output gap), the central bank given the task of minimizing the social loss function will be willing to pay a high price to make the policy robust to distortions (misspecifications) in the model. The reason for the concern for misspecification is that inflation typically reacts with a lag to monetary policy and there will be many possibilities of model misspecifications within the control lag. This may increase the social loss significantly in the most likely description of the model economy. The increase in loss might outweigh the inefficiencies of other policy regimes in which there are less uncertainty about the determinants of the objective variables and there will be less need for robustness. In particular, there would be no uncertainty associated with implementing a (base) money growth target regime since the objective is targeted perfectly by the central bank. Note that our argument does not depend on whether the central bank have access to commitment mechanisms. Even when the central bank can commit, model uncertainty influences the delegation problem.

Our results suggest that if the central bank doubts its model sufficiently, yet not to an implausible degree, delegating the Friedman rule of targeting the base money growth will be an improvement over inflation targeting in terms of social loss in the most likely description of the economy. Introducing imperfect control of money growth does not change this conclusion.

2. Model, policy preferences and robust control

We assume that the best available description of the monetary transmission mechanism is given by a New-Keynesian model. The work-horse log-linearized New-Keynesian model is derived in Woodford (2003) and is given by

$$\pi_t = \beta \pi_{t+1|t-1} + \gamma x_{t|t-1} + \varepsilon_t \tag{1}$$

$$x_t = x_{t+1|t} - \sigma \left(i_t - \pi_{t+1|t} - r_t^n \right) \tag{2}$$

where $\pi_t \equiv p_t - p_{t-1}$ is the inflation rate, x_t is the output gap, r_t^n is the natural real interest rate, and ε_t is a cost-push shock. Equation (1) is a forward-looking Phillips curve which allows for delayed effects of monetary policy on inflation using a one-period implementation lag. Equation (2) is an expectational IS curve. Both equations are structural and derived from microfoundations, see Roberts (1995), Woodford and Rotemberg (1997) and McCallum and Nelson (1999). The properties of the model have been analyzed extensively by Clarida et al. (1999) and Woodford (2003).

In order to relate the interest rate to base money, m_t, we introduce a money market. We specify the money demand equation as

$$m_t - p_t = x_t - \kappa i_t + v_t, \tag{3}$$

where v_t is a velocity shock and κ is the interest semi-elasticity of money.

The disturbances to the model follows AR(1) processes, i.e.,

$$\varepsilon_{t+1} = \rho_\varepsilon \varepsilon_t + \hat{\varepsilon}_{t+1}, \tag{4}$$

$$r^n_{t+1} = \rho_r r^n_t + \hat{r}^n_{t+1}, \tag{5}$$

$$v_{t+1} = \rho_v v_t + \hat{v}_{t+1}. \tag{6}$$

The model is parameterized by following standard values from the literature. We set $\beta = 0.99$, $\gamma = 0.024$, $\sigma = 6$, $\rho_\varepsilon = 0.5$, $\rho_r = 0.35$, $\sigma_{\hat{\varepsilon}} = 0.01$ and $\sigma_{\hat{r}} = 0.0372$.[1] In order to parameterize the demand for money, we estimate equation (3) over the period $1980q1 - 2004q4$ on US data using ordinary least squares and allowing for serial correlation of order one in the error term.[2] We obtained an estimate of $\kappa = 0.43$ with a coefficient standard error of 0.11.[3] Moreover, we estimate an autocorrelation coefficient of the disturbance term to $\rho_v = 0.77$ and a standard error of the shock to $\sigma_{\hat{v}} = 0.0116$.

The model described in equations (1-6) is the *reference model* (see Giordani and Söderlind, 2004), which structure and parameterizations represent the best available knowledge of the transmission of monetary policy to the monetary policymaker.

2.1. Robust monetary policy

The policymaker doubts the reference model but is not able to specify a probability distribution over the potential misspecification errors. Instead, the policymaker wants to guard against misspecifications that would lead to severe outcome.[4] In order to describe general uncertainty surrounding the reference model, we adopt the standard approach from robust control literature (see Hansen and Sargent, 2003a) and augment the reference model with a vector of additive misspecification terms η_{t+1}. The model including potential

[1]The parameter values for β, γ, σ, ρ_r and $\sigma_{\hat{r}}$ are taken from Giannoni and Woodford (2005, Section 1). The remaining parameters are set at values that do not seem a priori unreasonable.

[2]We used HP-filtered data on log monetary base, log GDP deflator, log GDP and the federal funds rate. The detrending was carried out with a smoothing parameter of 1600 over the period 1960q1-2004q4 to reduce the endpoint problem associated with the filter.

[3] Varying the endpoint of the estimation period between 1990q1 and 2005q4, produced estimates between 0.35 and 0.45, well within the range suggested by the standard error of the estimate. The estimate also stabilized very close to our point estimate after year 2000.

[4]For similar approaches in dealing with model uncertainty, see Hansen and Sargent (2003a,b), Giordani and Söderlind (2004), Dennis et al. (2005, 2006), Levin et al. (2003), Kilponen (2003, 2004), Levin and Williams (2003), Leitemo and Söderström (2004, 2005), Walsh (2004, 2005b,a), Onatski and Williams (2003) and Tetlow and Muehlen (2001).

misspecifications terms can be set up in state-space form as

$$\begin{bmatrix} x_{1,t+1} \\ \\ E_t x_{2,t+1} \end{bmatrix} = A \begin{bmatrix} x_{1,t} \\ \\ x_{2,t} \end{bmatrix} + Bi_t + C\left(\eta_{t+1} + \xi_{t+1}\right), \tag{7}$$

where A and B are matrices of model parameters, C is a vector that scales the impact of the vector of error terms ξ_{t+1}, $x_{1,t}$ is the vector of predetermined variables with $x_{1,0}$ given and $x_{2,t}$ is a vector of forward-looking variables.

The misspecifications are assumed to be bounded as

$$E_0 \sum_{t=0}^{\infty} \beta^t \eta_{t+1}' \eta_{t+1} \leq \eta_0, \tag{8}$$

where η_0 reflects specifically how large the potential misspecifications can be.

Monetary policy is delegated to an independent central bank that under commitment minimizes a period loss function that reflects the delegated objectives in a robust manner. Under the Friedman rule, the central bank's period loss function is simply $L_t = (\Delta m_t)^2$ while under inflation targeting loss function is the social loss function. Woodford (2003) shows that a quadratic approximation of economic welfare in the above reference model can be measured by the period social loss function

$$L^s = \left(\pi_t^2 + \frac{\gamma}{\psi} x_t^2\right), \tag{9}$$

where ψ is the elasticity of substitution between alternative differentiated goods. Following Woodford (2003), it is parameterized as $\psi^{-1} = 0.13$.

The policymaker assumes that misspecifications are of the worst kind and maximizes policy loss, subject to the constraint (8). Hansen and Sargent (2003a) and Giordani and Söderlind (2004) show that this problem can be stated as

$$\min_{i_t} \max_{\eta_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(L_t - \theta \eta_{t+1}' \eta_{t+1}\right)$$

subject to (7). θ summarizes the central bank's attitude towards model misspecifications in setting its policy. In particular, $\theta > 0$ relates to η_0 in such way that in the case with no misspecifications allowed $\lim_{\eta_0 \to 0} \theta = \infty$, while a smaller θ allows for greater misspecifications. A policymaker that is confident about the reference model will typically choose a high θ as he does not want to pay a high price in terms of social costs by deviating significantly from the optimal policy in a model he believes in.

The problem solves for the optimal choice of i_t and η_{t+1}. The equilibrium in the *worst-case model* can be described by substituting these solutions into (7) and then solving for the reduced form in the usual manner. The resulting system describes the worst-case model the central bank and the private sector want to guard against. We derive the approximating equilibria by assuming that there are no misspecification errors $\eta_t = 0$ for all t, but retaining the robust policy and expectations formation under the worst-case model. This describes equilibrium dynamics under robust decisions making by the private sector and the policymaker.

In order to calibrate θ for the present study, we adopt the concept of detection-error probability described by Anderson et al. (2003).[5] In this setup, a detection-error probability is the probability that an econometrician observing equilibrium outcomes would make an incorrect inference about whether the data was generated from the approximating equilibrium or the worst-case equilibrium. Smaller values for θ allows for greater specification errors, which make it easier for the econometrician to statistically distinguish between the two possible equilibria. Hence, a smaller θ reduces the detection-error probability.

2.2. Misspecifications in the worst-case model

The reduction in confidence about the model induces the policymaker to pay increased attention to model uncertainty and allows for the possibility of greater misspecifications in the model (η_0 increases $\to$ θ and the associated detection-error probability decrease). The monetary policymaker can only influence inflation with a lag and cannot respond to the cost-push shocks in the current period. The evil agent therefore introduces misspecifications by magnifying the cost-push shocks through making them more persistent than in the reference model. Figure 1 illustrates this by showing how the cost-push shocks

[5] In computing the detection-error probabilities, we follow Hansen et al. (2002).

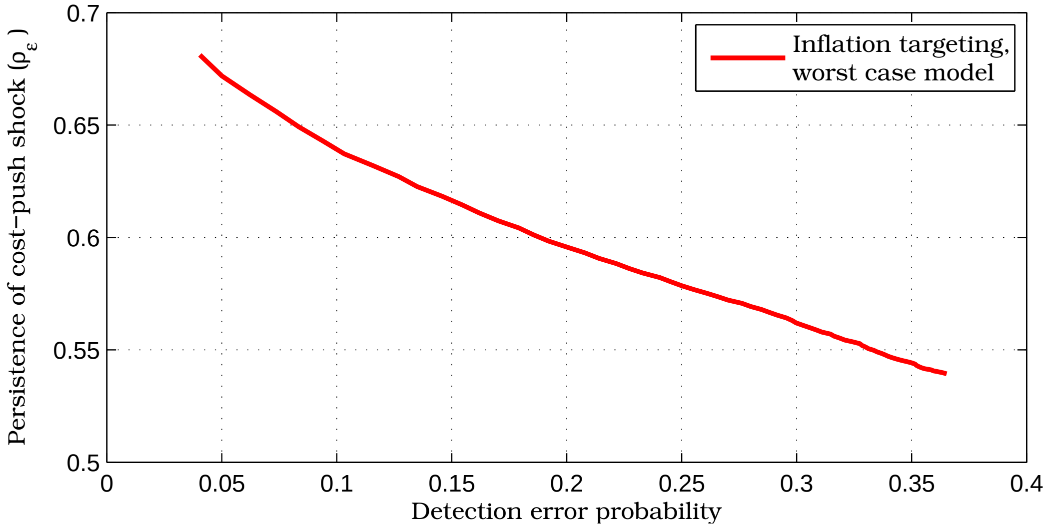

Figure 1: The figure shows the negative relationship between the implied persistence of the cost-push shock in the worst-case model and the detection-error probability under inflation targeting. The inflation-targeting central bank fears that cost-push shocks are more persistent than in the reference model where $\rho_\varepsilon = 0.5$.

become more persistent as the confidence in the model declines. The inflation-targeting designs the robust policy accordingly; the output gap will be more depressed in response to a positive cost-push shock. In the case of money growth targeting, the central bank controls the money base directly and thus it is not concerned with misspecifications in inflation and output determination. Money growth targeting central bank does not pay any price for robustness by deviating from policy that was optimal under most likely descriptions of the economy.

2.3. Comparing the regimes

The worst-case dynamic equilibrium is not the most likely outcome. The model without misspecification errors (reference model) is by construction the policymaker's best attempt at describing the monetary transmission mechanism. As noted above the approximating model assumes there are no misspecification errors $\eta_t = 0$ for all t, but retains the robust policy and expectations formation under the worst-case scenario. In Figure 2, we compare the expected social loss in the two regimes in both approximating and worst-case equilibrium at different degrees of preferences for robustness as represented by the detection-error probabilities.

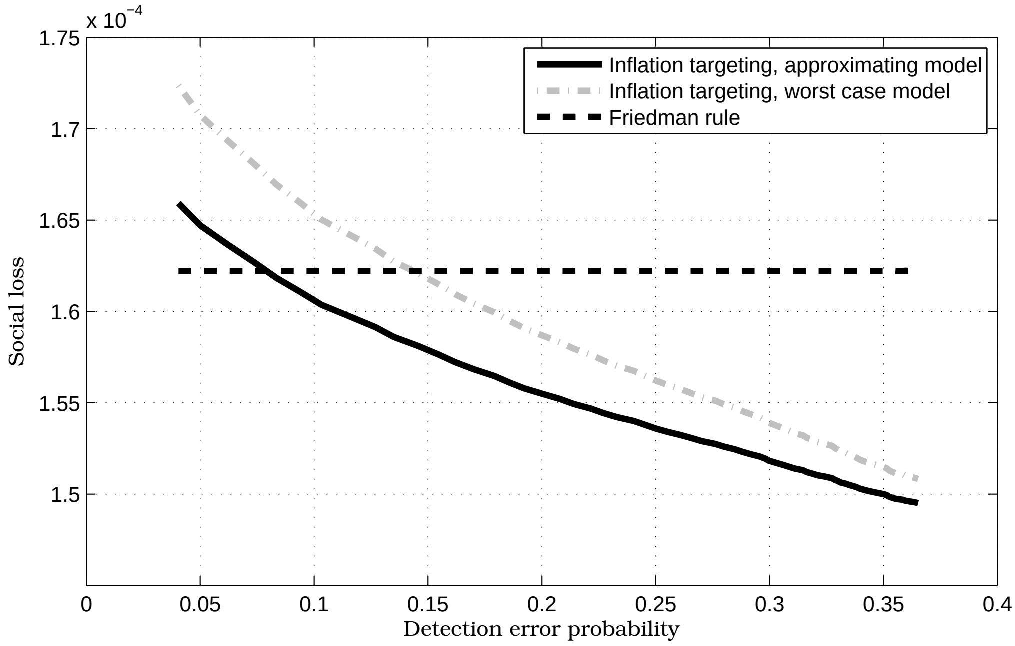

Figure 2: The figure shows the social loss (9) under different assumptions about preference for robustness, represented by detection-error probabilities, under inflation targeting and money growth targeting (Friedman rule).

If the policymaker has complete confidence in the model, the rational expectations equilibrium with the optimal solution to policy in the reference model applies. In this case, inflation targeting outperforms money growth targeting due to the usual arguments. The policymaker has preferences that are similar to those of society. Consequently, given a correct description of monetary policy transmission mechanism, the policymaker will minimize social loss. Money growth targeting is moreover subject to the inefficiency due to persistent velocity shocks.

Now consider the case of declining confidence in the reference model. As noted above, the central bank designing robust policy in order to respond to the potential worst-kind misspecifications described above. The cost of this is that the outcome in the case where the reference model is the true model is worsened [described by the approximating equilibrium]. This is the price paid for robustness under inflation targeting.[6] As discussed earlier, under money growth targeting, the central bank controls money perfectly and has no incentives to pay a price for insurance against possible misspecifications in the model.

[6]Figure 2 also plots the social loss in the worst case equilibrium. This is higher than in the approximating equilibrium due to the actual inclusion of misspecification errors.

The conclusion is that if a policymaker has small confidence in his description of the transmission mechanism, it would do better by adopting the Friedman rule. In our calibration, the threshold level is roughly at detection error probability of 8% for the approximating equilibrium. These values suggest that Friedman's k-percent rule becomes attractive when the model uncertainty is relatively large, yet not implausibly so.

2.4. Imperfect monetary control

Equation (3) suggests that the money growth can be perfectly controlled by the central bank. While this may be correct in the case of a very narrow money aggregate, monetary authority has incomplete control over the broader definitions of money. If the central bank chooses to target a broader definition, it would typically target the money aggregate through the setting of the short-term interest rate. It can then be shown that in the New Keynesian model imperfect monetary control does not jeopardize the conclusion above? In order to see this, consider the following case where the money demand is written in differenced form:

$$dm_t = dp_t + dx_t - \kappa di_t + dv_t. \tag{10}$$

Moreover, assume that the change in the velocity shock is only imperfectly observed by the policymaker when setting the interest rate. In particular, we write that

$$dv_t = dv_t^o + dv_t^u, \tag{11}$$

where the CB observes dv_t^o but know that dv_t^u follows a white noise process.

The central bank targets money growth given from equation (10). Money growth can be decomposed into two parts: The part that CB controls i.e. the optimal forecast of the money growth, given by

$$dm_{t|t} \quad = \quad dp_t + dx_t - \kappa di_t + dv_t^o, \tag{12}$$

and the part it has no influence over, i.e., the unobserved velocity shock dv_t^u. Thus money growth is given as $dm_t = dm_{t|t} + dv_t^u$.

In the reference model, the central bank sets the interest rate so as to have a forecast of the growth rate money equal to $k\%$. The forecast error variance is now equal to the variance of the unobserved velocity shock. In the worst-case equilibrium the money demand equation is distorted by the magnification of the unobserved velocity shock. This increases the money growth forecast error and the central bank loss in the worst-case equilibrium. However, this does not lead to increase in the social loss, neither in the worst-case nor in the approximating equilibrium. The reason is that unobserved velocity shock does not influence inflation and output. It is a well known feature of the New Keynesian model, that money does not influence these variables directly, but works through the interest rate channel only. Moreover,the central bank does not respond to increased forecast error variance because the interest rate is set in accordance with the forecasted velocity shocks and there is no way they can reduce the forecast error variance. The only difference with respect to perfect money growth control analyzed previously is that the loss of the central bank is increased in the worst-case equilibrium. Consequently, introducing imperfect control over money growth does not alter the conclusion about delegation in the paper.

3. Conclusions

Our analysis suggests that the presence of model uncertainty should be taken into account in the delegation of monetary policy to an independent central bank. Setting advanced objectives for an independent monetary authority can be welfare reducing if the central bank has insufficient confidence in its ability to precisely target the objective variables and therefore needs to undertake costly insurance in the form of robust policy design. It may then be welfare increasing to delegate a less ambitious target to the central bank which the central bank is more confident in controlling.

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