Targeting Inflation by Forecast Feedback Rules in Small Open Economies*

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Abstract

We argue that in practice, the inflation-targeting strategy can be approximated by the interest rate responding to the unchanged-interest-rate forecast of inflation. A method is developed to derive unchanged-interest-rate forecasts in forward-looking models and evaluate the performance of the policy rule in an optimizing New Keynesian model due to Monacelli (2003), estimated on UK data. We find that the policy rule is less prone to generate a determinate rational expectations equilibrium if based on an unchanged interest rate, compared to the rule-consistent forecast. Both rules approximate the optimal commitment policy if the central bank attaches sufficient weight to inflation as opposed to output gap stabilization. The optimal forecast-feedback horizon is close to a year and a half and is largely independent of how much the central bank prefers inflation to output gap stability.

Keywords: Monetary policy, inflation targeting, feedback rules, small open economy.
JEL codes: E61, E47, E43.

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1. Introduction

The arguably poor performance and robustness of fixed exchange rate systems and monetary targeting have resurrected a belief in more activist policy throughout the 1990s. Such activism is normally associated with the central bank’s discretionary use of the interest rate in order to directly steer policy toward price stability, in the sense of low and stable inflation. Such a framework is often referred to as inflation targeting. Inflation targeting has been formally introduced in several countries, e.g., New Zealand, Canada, Sweden, the United Kingdom, Australia, Norway and Iceland, where the central banks have been given explicit targets for inflation and instrument independence to set the interest rate so as to achieve the inflation target.

We interpret inflation targeting as adherence to a forecast feedback rule for the interest rate where the deviations of the forecast of inflation from the target level are the prominent indicator. If the inflation forecast is above (below) the inflation target, the central bank sets a contractionary (expansionary) monetary policy stance, i.e., by setting the interest rate above (below) its natural rate or moving the interest rate in steps towards this target rate. This interpretation is in line with the interpretations made by Batini and Haldane (1999), Batini and Nelson (2001), Levin et al. (2003) and others.1 Several central banks state the use of such a procedure to guide policy. Sveriges Riksbank (1999) Inflation Report 3/99, p. 58 states:

Monetary policy is sometimes described with a simple rule of thumb: if the overall picture of inflation prospects (based on an unchanged repo rate) indicates that in twelve to twenty-four months’ time inflation will deviate from the target, then the repo rate should normally be adjusted accordingly. (My italics)

Jansson and Vredin (2003) interpret the procedure of monetary policymaking at Sveriges Riksbank as the use of forecasts feedback rules.

Svein Gjedrem, the Governor of the Central Bank of Norway, states

The key rate is set on the basis of an overall assessment of the inflation outlook two years ahead. If it appears that inflation will be higher than 2 per cent with unchanged interest rates, the interest rate will be increased. If it appears that inflation will be lower than 2 per cent with unchanged interest rates, the interest rate will be reduced. (Gjedrem, 2002) (My italics)

A representation of such a forecast feedback rule is given by

\[ r_t = \rho r_{t-1} + (1 - \rho) \beta \left[ \hat{\pi}_{t+H} - \hat{\pi}^* \right], \]  

(1)

1 An alternative interpretation is offered by Leitemo (2006), where I study the effects of setting the interest rate so as to have the constant-interest-rate forecast of inflation equal to the target at some given horizon.
where $r$ is the policy interest rate, $\bar{\pi}^*$ the annual inflation target and $\hat{\pi}_{t+H}$ the H-period-ahead forecast of the annual inflation rate. $H$ is the forecast-feedback horizon, which is distinguished from the policy target horizon, i.e., the expected time before inflation has returned to its target level (see also Batini and Nelson, 2001). The rule allows for interest rate smoothing, i.e., the partial adjustment of the interest rate, which may be important for the central bank to have a beneficial influence on private sector behavior by affecting private agents expectations about future policy (see Woodford, 2003b).

Although research on forecast feedback rules, which is discussed below, has almost exclusively been based upon the assumption of rule-consistent forecasts, i.e.,

$$\hat{\pi}_{t+H} = E_t[\bar{\pi}_{t+H}],$$

central banks have typically used an unchanged-interest-rate assumption (see, e.g., the italicized text in the quotations) in deriving the inflation forecasts, that is, deriving expected inflation conditional on the interest rate not being changed throughout the forecast-feedback horizon, i.e.,

$$\hat{\pi}_{t+H} = E_t[\bar{\pi}_{t+H}|\bar{r}_{t-1}].$$

We use the abbreviation PCF for denoting the policy-consistent forecast-feedback rule (equations (1) and (2)), and UIF for denoting the unchanged-interest-rate forecast-feedback rule (equations (1) and (3)). The forecast assumption is important when the forecast-feedback horizon is longer than the policy control lag and the inflation forecast does not only depend on the present policy stance, but also on the future policy stance. Although the assumption of a policy-consistent interest rate throughout the forecast horizon ensures consistency, it may be somewhat unrealistic from a practical point of view. Forecasts based on assumptions about specific future interest rate changes may be of little guidance to the interest rate decision body that may have a hard time just deciding about the present interest-rate stance. Svensson (1999a) argues that the forecast should be based on an unchanged interest rate, which allows the decision body to focus on the current interest rate setting, and not having to form expectations about future interest rate decisions. Moreover, Svensson argues that it may be easier to incorporate outside-of-the-model information under such a procedure, since such information may take the form of the policymakers’ judgment conditional on the policy stance remaining unchanged.

So far, however, there has been no procedure for handling the unchanged-interest-rate assumption in models where agents display forward-looking behavior. This paper presents a

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2 The forecast-feedback horizon and the policy target horizon will only coincide when the length of the horizon is such that inflation will have returned to the target rate of inflation without any reactions by the monetary authority to the state of the economy, i.e., the interest rate is kept constant at its equilibrium value. In this case, $\pi_{t+H} = \pi^* = \pi^*$, and $i_t = i = r + \pi^*$, where superscript $e$ denotes an equilibrium value and $r$ is the short-term real interest rate.

3 The analysis by Rudebusch and Svensson (1999) of forecast feedback rules in a backward-looking model of the US economy is an important exception.
method for doing exactly that. Using this method, we evaluate the difference between the two forecast assumptions using an empirical optimizing New Keynesian model of the UK economy. An important result is that we find the difference between the PCF and UIF rules to be small, given that the forecast-feedback horizon is short. It becomes more important at horizons exceeding six quarters. Although both assumptions make the rule prone to rational expectations indeterminacy at long forecast horizons, the assumption of an unchanged-interest-rate forecast increases the region of rational expectations indeterminacy. We find that both the UIF and PCF rules are successful at stabilizing inflation but less so at stabilizing the output gap, thus supporting and extending the result in Rudebusch and Svensson (1999) and Levin et al. (2003) to the small open economy. The optimal forecast-feedback horizon is found to be around a year and a half, irrespective of how strict the central bank is on inflation stabilization. This length is close to the forecast-feedback horizon used by many inflation-targeting central banks.

PCF rules have been extensively discussed in the literature. Batini and Haldane (1999) argue that the rule is “lag encompassing”, i.e., takes account of the fact that monetary policy works with a lag on inflation by focussing on the inflation forecast. By responding to the forecast of inflation at a sufficiently long length, it ensures that policy preemptively responds to those inflationary shocks monetary policy may indeed counteract. The policy rule includes the inflation forecast as an indicator and therefore, embodies all relevant knowledge about future inflation. The rule is therefore “information encompassing”. Finally, they show that within a small forward-looking macroeconomic model, the rule is successful at stabilizing both inflation and output (i.e., rule is “output encompassing”) without causing too strong movements in the interest rate. Batini and Nelson (2001) evaluate the rule in both a vector autoregressive (VAR) model and a small forward-looking macroeconomic model and find that the optimized rule performs close to the optimal commitment policy. The optimal forecast-feedback horizon, however, depends greatly on the particular model, being two quarters for the forward-looking model and as long as fifteen quarters for the VAR model. Rudebusch and Svensson (1999) compare the performance of several rules for inflation targeting in a backward-looking model of the US economy and find that the forecast-feedback rules perform close to the optimal rule, as long as the central bank does not put a relatively large weight on stabilizing output. Their study suggests the performance to be relatively independent of the choice of forecast feedback horizon, as long as it is beyond two years. Levin et al. (2003) study forecast feedback rules in five models of the US economy and also find, as noted above, that the rule is successful at stabilizing inflation but lacks some of the output encompassing features found by Batini and Haldane (1999). However, by extending the rule to include the output gap as an indicator and responding to the one-year-ahead inflation forecast, an appropriately calibrated rule does not only more efficiently stabilize output, but also becomes more robust to model uncertainty, i.e., it works well in all five models. They find that rules with a longer forecast feedback horizon are prone to rational expectations indeterminacy, and also less robust to model uncertainty.
The remainder of the paper is organized as follows: Section 2 discusses the use of the unchanged-interest-rate assumption in constructing inflation forecasts. Section 3 presents a New Keynesian model of a small open economy due to Monacelli (2003) with both domestic goods producers and firms importing goods from abroad experiencing rigidities in price setting. Section 4 presents the stabilization and determinacy properties of the two types of policy rules and discusses the best choice of the forecast-feedback horizon. Finally, Section 5 provides the main conclusions.

2. Constructing unchanged-interest-rate forecasts in forward-looking models

It is important to note that the unchanged-interest-rate assumption is merely an assumption invoked in the construction of the inflation forecast. It does not imply that the interest rate in general is expected to remain unchanged throughout the forecast-feedback horizon under the UIF rule. The interest rate is expected to follow the rule prescription, not the assumption. The rule would only prescribe an unchanged interest rate in very rare cases.

The construction of unchanged-interest-rate forecasts of inflation in models with forward-looking variables requires some comments. In particular, a clarification is needed on how the forward-looking agents use the unchanged interest rate assumption in deriving their own forecasts which are important for the determination of the forward-looking variables. Since private agents forms expectations rationally, they expect that the interest rate will be set according to the rule and thus, will normally be changed. They also know, however, that the kind of assumption made in deriving the inflation forecasts will be of importance for the interest rate prescription of the rule and thus, for the current and expected future setting of interest rates.

The policymaker knows that rule-based interest rate setting does not reveal any new information about the future to the private agents, he takes the forward-looking variables as predetermined when constructing the unchanged-interest-rate forecasts. In forecasting the forward-looking as well as the predetermined variables influencing inflation, however, the interest rate is assumed to remain unchanged throughout the forecast-feedback horizon. Hence, the unchanged-interest-rate forecasts are partly based on the policy rule itself (in determining current forward-looking variables), and partly on the unchanged-interest-rate assumption in determining the forecasts. This is the inconsistency inherent in the UIF rules. Appendix A gives a formal treatment of how to incorporate the UIF rule in forward-looking models with a state-space representation.4

4A description of how to incorporate a PCF rule in a state-space representation of the current forward-looking model is available from the author upon request. See also Rudebusch and Svensson (1999) for the representation of the PCF rule in a backward-looking model.
3. A New Keynesian model with imperfect exchange-rate pass-through

To evaluate the inflation forecast-feedback rules, a model of private sector behavior is required. The New Keynesian model framework (see, Woodford (2003a) and Clarida et al. (1999)) introduces a role for monetary policy in alleviating the assumed price stickiness through proper stabilization policy. The framework gives rise to a tractable system of two equations representing aggregate supply and demand. This framework has been extended to the open economy by Clarida et al. (2002), Gali and Monacelli (2004) and others. Clarida et al. (2001) argues that the closed and open economy versions of the model are isomorphic, as also the open economy framework can be reduced to this tractable system of two equations. As pointed out by Monacelli (2003), however, this isomorphism is broken once the domestic currency price of imported goods is subject to incomplete pass-through and there are deviations from the law of one price. Rogoff (1996) and Goldberg and Knetter (1997) suggest that deviations from the law of one price are important and persistent. Since our purpose is to evaluate rules for inflation targeting, it is important to account for as much of the inflation dynamics as possible and the Monacelli model is therefore attractive.

For convenience, we give a brief presentation of the log-linearized version of the Monacelli model, with the coefficients being policy-invariant functions of utility and technology parameters. For details, the reader is referred to Monacelli (2003).

All domestic firms operate in a setting of monopolistic competition and want to set prices as a markup on marginal costs. Prices are, however, subject to stickiness of the Calvo (1983) type. For domestic producers, the aggregate supply is then described by a New Keynesian Phillips curve of the form

\[ \pi_H^t = \beta E_t \pi_H^{t+1} + \kappa_x x_t + \kappa_{\psi} \psi_{F,t} \]  

where \( \pi_H^t \equiv p_H^t - p_{H,t-1} \) is the rate of price inflation on goods domestically produced in period \( t \), \( x_t \) is the output gap, the percentage deviations of output from the flexible-price level of output, and \( \psi_{F,t} \) is the Law-of-One-Price (LOP) gap, i.e., the percentage deviation of the world market prices (measured in terms of domestic currency) from the domestic price of foreign goods,

\[ \psi_{F,t} \equiv e_t + p^*_t - p_{F,t} = q_t - (1 - \gamma)s_t. \]

Here, \( e_t \) is the nominal exchange rate, \( p^*_t \) the world market price measured in foreign currency,

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5 The open economy Phillips curve (4) illustrates the point made in Monacelli (2003), i.e., that the deviations from the law of one price have a influence on inflation, separate from the influence of the output gap, and complete inflation stability can no longer be achieved through complete stabilization of the output gap. The deviation is a way of representing the “cost-push” shocks often added in an add-hoc way to the Phillips curve to introduce a trade-off between inflation and output gap stabilization.
the domestic price of foreign goods (imported goods price), 

\[ q_t \equiv e_t + p_t^* - p_t \]

the real exchange rate, and \( s_t \) the terms of trade, i.e., the price deviation between the domestic price of foreign goods and the price of the domestic goods,

\[ s_t \equiv p_t^F - p_t^H. \] (6)

The nominal exchange rate is determined by the uncovered interest rate parity condition, assuming perfect capital mobility,

\[ e_t = E_t e_{t+1} - r_t + r_t^*, \] (7)

where \( r_t \) and \( r_t^* \) are the domestic and foreign short-term nominal interest rates, respectively.

The economy is populated by infinitely-lived households that optimize utility over time by consuming both domestic and imported goods. The demand for the domestic product, represented by the output gap, is given by

\[ x_t = E_t x_{t+1} - \chi(r_t - E_t \pi_{t+1}^H - \pi_t) + \varsigma E_t (\Delta \psi F_t + \pi H_t), \] (8)

where the natural real interest rate, \( \pi^r_t \), is given by

\[ \pi^r_t \equiv \phi E_t \Delta y^*_t + \theta z_t, \] (9)

where \( z \) is a domestic productivity shock and \( \Delta y^* \) the rate of change in world output.

The importers of the foreign goods want to set prices as a markup on the world market price, but their prices are subject to stickiness. The stickiness in imported goods prices gives rise to an incomplete pass-through as prices gradually move towards world market prices. The supply curve becomes

\[ \pi^F_t = \beta E_t \pi^F_{t+1} + \lambda \psi F_t. \] (10)

CPI inflation, \( \pi_t \equiv p_t - p_{t-1} \), is a weighted average of domestic and foreign goods price inflation,

\[
\pi_t = (1 - \gamma) \pi_{t+1}^H + \gamma \pi^F_t \\
= \pi_t^H + \gamma \Delta s_t, \] (11)

where \( \pi^F_t \equiv p_t^F - p_{t-1}^F \) is the rate of price inflation on imported goods and \( \Delta s_t \equiv \pi^F_t - \pi^H_t \) is the rate of change in the terms of trade.
3.1. The empirical specification

Although the above theoretical framework gives a canonical representation of private-sector behavior in an economy where goods prices are subject to stickiness, the framework abstracts from possible information and implementation lags that may give rise to gradual adjustment in the real world. Such inertial responses may be rationalized and explained by agents using rule-of-thumb pricing (Christiano et al., 2005), and consumers being subject to habit formation (Fuhrer, 2000). For these reasons, we follow Rudebusch (2002a,b) in allowing data to influence the structure of leads and lags in the economy and add an assumed white-noise error term to each estimated equation. In general, we start out by a general lag structure with four lags of the endogenous variable and then reduce the structure by eliminating insignificant lags, starting with the least significant one. However, we do not eliminate insignificant lags important for the residuals to be relatively free of serial correlation. As in Rudebusch (2002b), we use the expected annual inflation over the next year to represent the forward component of inflation in the Phillips curve, assuming that prices, on average, are changed once a year. The decisions are subject to a one quarter implementation lag. Moreover, all equations are estimated with a (non-reported) intercept term. The model is estimated on UK data obtained from either the national accounts or the IMF and OECD databases.

The Phillips curve for domestic inflation is estimated as

\[
\pi_{t+1}^H = \mu_H E_t \bar{\pi}_{t+4}^H + (1 - \mu_H) \sum_{j=0}^{3} \alpha_j \pi_{t-j}^H + \kappa_x E_t x_{t+1} + \kappa_\psi E_t \psi_{t+1} + \varepsilon_{t+1},
\]

where \(\pi_t^H \equiv 4(p_t^H - p_{t-1}^H)\) is the quarterly percentage increase in the GDP deflator measured as an annual rate, and \(\bar{\pi}_t \equiv \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}\) is the four-quarter inflation rate. The estimation period is 1980Q1 - 2001Q4 and the model is estimated by GMM. We impose dynamic homogeneity, i.e., \(\sum_{j=0}^{3} \alpha_j = 1\). The LOP gap has been computed according to equation (5), using a detrended effective real exchange rate and terms of trade.\(^6\) The terms of trade were derived as the percentage deviation between the imported goods prices and the domestic price level. The share of imported goods in the consumer basket is set at \(\gamma = 0.25\).\(^7\) The output gap is detrended log GDP. Expected future inflation, \(E_t \bar{\pi}_{t+4}^H\), was instrumented using eight lags of the quarterly domestic inflation rate, fours lags of the deviations from the law of one price, the output gap, the UK 3-month interest rate, the US federal funds rate and the OECD output.

\(^6\) All detrending was performed using a Hodrick-Prescott filter with the smoothing parameter set at 1600.

\(^7\) This corresponds to the value used in Batini and Haldane (1999) and is reasonable for a small open economy.
The preferred equation\(^8\) is given by

\[
\pi_{t+1}^H = 0.58 E_t \pi_{t+4}^H + 0.42 \left( -0.39 \pi_{t-1}^H + 0.22 \pi_{t-2}^H + 0.72 \pi_{t-3}^H - 0.45 \pi_{t-4}^H \right) + 0.28 E_t x_{t+1} + 0.04 E_t \psi_{t+1}^F + \varepsilon_{t+1}
\]

\[R^2 = 0.67 \quad \sigma = 0.02 \quad DW = 1.60.\]

We note that the data prefers a slightly higher weight on the forward component as compared to the backward component. The proposition that the weights are equal cannot be rejected, however. Moreover, we find endogenous inflation persistence with a considerable lag length. Both the output gap and the LOP gap estimates have the sign that should be expected from theory.

Imported goods price inflation is estimated according to the form

\[
\pi_{t+1}^F = \mu_F E_t \pi_{t+4}^F + (1 - \mu_F) \sum_{j=0}^{3} \kappa_j \pi_{t-j}^F + \omega \psi E_t \psi_{t+1}^F + \nu_{t+1}, \quad (14)
\]

where \(\pi_t^F \equiv 4(p_t^F - p_{t-1}^F)\) is quarterly imported goods price inflation measured as an annual rate, and \(\bar{\pi}_t^F \equiv \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j}^F\) is the four-quarter imported goods inflation rate. The model was estimated over the period 1980Q1 – 2001Q4 using GMM. The instruments are eight lags of imported goods price inflation and four lags of the LOP gap, the output gap, the UK 3-month interest rate, the US federal funds rate and the OECD output gap. The preferred model is then given as

\[
\pi_{t+1}^F = 0.78 E_t \pi_{t+4}^F + 0.22 \left( 1.11 \pi_t^F - 0.11 \pi_{t-3}^F \right) + 0.56 E_t \psi_{t+1}^F + \nu_{t+1} \quad (15)
\]

\[R^2 = 0.46 \quad \sigma = 0.06 \quad DW = 1.92.\]

Imported goods inflation seems to be following a significantly more forward-looking process than domestic inflation, and there is considerably less endogenous inflation persistence. This is evidence consistent with a smaller share of price setters that follows backward-looking pricing rules in this sector. We do note, however, that the goodness of fit of the equation is considerably worse as compared to that in the case of domestic inflation, thereby suggesting that the equation might not pick up all factors influencing imported goods prices.

\(^8\)The standard error of the estimates is stated in parenthesis under the estimate, \(R^2\) states the percentage variation in the endogenous variable that is explained by the explanatory variables, \(\sigma\) is the standard error of the residuals and \(DW\) is the Durbin-Watson statistics for autocorrelation in the residuals.
The output gap is estimated as

\[
x_{t+1} = \mu_x E_t x_{t+2} + (1 - \mu_x) \left( \eta x_t + (1 - \eta) x_{t-1} \right) - \chi (r_t - \bar{E}_t \bar{\pi}_H^{t+3}) + \xi E_t \Delta \psi^F_{t+1} + \phi E_t \Delta y^*_t + u_{t+1},
\]

where \( y^* \) is foreign output approximated by the OECD output gap. The future expected output gap was instrumented using four lags of quarterly domestic inflation rate, the LOP gap, the output gap, the UK 3-month interest rate, the US federal funds rate and the growth rate of OECD GDP. The preferred equation is given by

\[
x_{t+1} = 0.53 E_t x_{t+2} + 0.47 \left( \frac{1.36 x_t - 0.36 x_{t-1}}{0.08} \right) - 0.07 (r_t - \bar{E}_t \bar{\pi}_H^{t+3}) + 0.11 E_t \Delta \psi^F_{t+1} + 0.25 E_t \Delta y^*_t + u_{t+1}.
\]

The parameters in the output gap equation are more precisely estimated and the goodness of fit is greater than in the equations for inflation. The parameters have the correct sign; in particular the parameters in front of the real interest rate and the change in the LOP gap have small standard errors. The forward and backward-components are of similar size, and endogenous persistence has a shorter lag length than inflation.

The uncovered interest parity condition was estimated assuming that the unobserved risk-free foreign real interest rate can be approximated with an autoregressive process. Unconstrained estimation yields

\[
q_t = 0.997 E_t q_{t+1} - 0.965 (r_{q,t} - E_t \bar{\pi}_{q,t+1}) + r r^*_{q,t},
\]

\[
rr^*_{q,t} = 0.34 r r^*_{q,t-1} + w_t
\]

where \( r_{q,t} \equiv \frac{1}{4} r_t, \pi_{q,t} \equiv \frac{1}{4} \pi_t \) and \( rr^*_{q,t} \equiv \frac{1}{4} rr^*_t \) are the UK 3-month interest rate, the quarterly CPI inflation rate, the foreign real interest rate, respectively; all measured as quarterly rates. The instruments are four lags of the real effective exchange rate, the UK 3-month interest rate, the US federal funds rate, the quarterly CPI inflation rate and the OECD output gap. The residuals were found to be well modelled by an AR(1) process as additional lags were insignificant. Although imprecisely estimated, the interest rate term has a coefficient almost equal to the theoretical expected value of unity. Similarly, the coefficient on the forward exchange rate term is also almost unity, as expected from theory. By constraining the coefficients to unity, the
preferred model is given by
\begin{align}
q_t &= E_t q_{t+1} - (r_{q,t} - E_t \pi_{q,t+1}) + r^{*}_{q,t} \\
nr^{*}_{q,t} &= 0.50 rr^{*}_{q,t-1} + 0.19 rr^{*}_{q,t-2} + 0.11 rr^{*}_{q,t-3} + w_t \\
R^2 &= 0.85 \quad \sigma = 0.037 \quad DW = 2.12,
\end{align}
and the foreign real interest rate is best approximated by an AR(3) process.

Finally, the OECD output growth is modelled according to an autoregressive process as
\begin{align}
\Delta y^*_t &= 0.51 \Delta y^*_{t-1} + \xi_t \\
R^2 &= 0.25 \quad \sigma = 0.005 \quad DW = 2.14.
\end{align}

4. Policy analysis

In this section we start by giving an account of the monetary policy objectives. We then proceed to give a description of the transmission mechanism of the model by considering a disinflationary experiment, unexpectedly reducing the inflation target by one percentage point, assuming that policy is implemented through the forecast-feedback rule. Then, the configuration of the rule parameters is considered from two perspectives. First, we infer what configurations (if any) of the rule yield a determinate rational expectations equilibrium, and, second, we infer among the rule configurations which yield a rational expectations equilibrium whether the rule may come close to replicating the outcome of the optimal commitment policy.

4.1. Monetary policy objectives

In the first two sections, we described the strategy of the central bank. We now turn to the objectives. We shall assume that the central bank has conventional inflation targeting preferences as described in Svensson (1997, 1999b, 2000) among others, i.e., in each period, it wants to minimize the squared output gap (i.e., the percentage deviation of actual output from the natural rate) and inflation deviations from the inflation target in addition to the change in the policy interest rate. The period loss function is given by
\begin{align}
L_t = (1 - \lambda)(\bar{\pi}_t - \bar{\pi}^*)^2 + \lambda x_t^2 + \nu(\Delta r_t)^2,
\end{align}
where $x$ is the output gap and $\lambda \in [0, 1]$ and $\nu \geq 0$ are the relative weights attached by the monetary policymaker to output versus the inflation stabilization objective and interest-rate smoothing, respectively. The central bank is assumed to minimize the expected sum of
Figure 1

The response to an unexpected reduction in the inflation target with $H = 8$.

The figures show that the response to key variables as the inflation target is reduced by one percentage point in combination with either the UIF or the PCF rule. In the case of the UIF rule, the inflation figures also show the evolution of inflation, if the interest rate remains unchanged throughout the forecast-feedback horizon.

The discounted period loss assuming that the discount factor ($\delta$) approaches unity,

$$\lim_{\delta \to 1} (1 - \delta) \sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau},$$

subject to the model of the monetary policy transmission mechanism. As shown in Rudebusch and Svensson (1999), the value of the intertemporal loss function as $\delta \to 1$ in (24) approaches the unconditional mean of the period loss function, which equals the weighted sum of the unconditional variances of the goal variables,

$$EL_t = (1 - \lambda) \text{var}(\bar{\pi}_t - \bar{\pi}^*) + \lambda \text{var}(x_t) + \nu \text{var}(\Delta r_t),$$

which we use as our loss function in the paper.

4.2. A disinflationary experiment

Figure 1 shows the response of key variables to an unexpected reduction in the inflation target for the UIF and PCF rules. We assume a forecast feedback horizon of eight quarters, which seems reasonable given the statements by the central bankers quoted in the introduction. Note that variables are measured relative to their new steady-state values. We assume that the policymaker sets $\rho_r = 0.95$ and $\beta_{\pi} = 6.25$ in case of the UIF rule and $\rho_r = 0.8$ and $\beta_{\pi} = 10$ for the PCF rules.\footnote{Section 4.4 show that these coefficients determine the rational expectations equilibrium and produce the least loss at the stated forecast-feedback horizon.}

The announcement of a lower inflation target makes private agents reduce their inflation...
expectations, which leads to an increase in the real interest rate. The unchanged-interest-rate forecast produces an undershooting of the inflation target at the relevant horizon and the nominal interest rate is gradually reduced. Lower expectations about future inflation implies expectations of a lower price level. Thus, the nominal exchange rate immediately appreciates and depreciates toward its new steady-state growth path. As there are expectations of future positive real interest rate (differentials), the real exchange rate appreciates at the time of the announcement and then depreciates at a rate equal to the real interest rate differential. The increase in the real interest rate and the temporary real appreciation reduce the output gap over a three-year period. After three years, CPI inflation is approximately back on target. Figure 1(b) shows the responses under a PCF rule to be quite similar, albeit somewhat stronger as the interest rate reacts somewhat more aggressively to the disinflation shock.

The similarity between the responses is dependent on the coefficients being optimally selected. If, for instance, we use the optimally selected parameters for the UIF rule in the PCF rule, the PCF rule with a feedback horizon of eight quarter produces overstabilization and cyclical evolvement in the model.\textsuperscript{10}

The disinflation experiment illustrates that the forecast-feedback horizon may deviate substantially for the policy target horizon; it always takes longer than the forecast-feedback horizon to bring inflation back to its target in steady state. The exact size of the deviation is, however, dependent on the configuration of the rule as well as the type of shock to which the economy is subject.

4.3. Determinacy

An interesting issue is whether the rules considered determine a unique equilibrium. If not, there may be multiple solutions to the model and expectations may be self-fulfilling. The importance of determinacy has been stressed by Woodford (2003a) and many others. If policy does not pin down unique solution paths of inflation and other variables, the economy is not only susceptible to endogenous cycles and sunspots (see, e.g., Clarida et al., 2000). Furthermore, the selection criteria for choosing which equilibrium is the more “relevant” is controversial.\textsuperscript{11} We will now consider whether the forecast feedback rules determine the rational expectations (RE) equilibrium. This issue earlier been discussed by Bernanke and Woodford (1997) who argue that forecast-based rules are susceptible to indeterminacy, and even to the non-existence of an equilibrium if based of private-sector forecasts. Levin et al. (2003) also discuss the determinacy and conclude that in particular for long forecast-targeting horizons, the feedback rule is prone to yield an indeterminate RE equilibrium. We arrive at the same conclusion.

\textsuperscript{10}The response figure is available from the author upon request.

\textsuperscript{11}The minimum state variable solution criterion of McCallum (see 1983, 1999) has, however, been put forward as a selection criterion and has some desirable properties, in particular, that it is learnable in many cases McCallum (see 2002). See Evans and Honkapohja (2001) for further information about the learnability of rational expectations.
Blanchard and Kahn (1980) show that for the RE equilibrium to be determined, there must be as many unstable eigenvalues as there are forward-looking variables in the model. This turns out to be an important issue for the forecast-feedback rules, as this requirement is not met for a large set of parameter values. Figures 2 and 3 show the configurations of the rule parameters \( \{H, \beta_\pi, \rho_r\} \) that produce a determinate RE equilibrium for the UIF rule and the PCF rule, respectively. There are at least three important observations to be made.

**Figure 2**
Parameter determinacy region for the UIF rule at different forecast-feedback horizons.

The figures show the combinations of \( \beta_\pi \) and \( \rho_r \) for different forecast-feedback horizons that yield a determinate rational expectations equilibrium. The blank region is the indeterminacy region.

First, the length of the forecast-feedback horizon is important for determinacy. In this study we considered forecast-feedback horizons up to twelve quarters. The parameter region of determinacy decreases as the horizon increases, and becomes very small as the forecast-feedback horizon is above eight quarters. The requirement for determinacy is often stated as the Taylor principle (Woodford, 2001), which means that the nominal interest rate needs to react sufficiently to increased inflation expectations to increase the real interest rate. Note that the real interest rate is determined by next-period inflation expectations, and a reaction to inflation expectations in the more distant future may not suffice to raise the real interest rate. Hence, a long forecast-feedback horizon may fail to determine the RE equilibrium. As noted
The figures show the combinations of $\beta_\pi$ and $\rho_r$ for different forecast-feedback horizons that yield a determinate rational expectations equilibrium. The blank region is the indeterminacy region.

above, this result is also found in Levin et al. (2003).

Second, a higher value of $\beta_\pi$ is likely to produce indeterminacy since the rule causes inflation to undershoot the target at the relevant horizon and a strong response is likely to reduce the nominal interest rate and therefore, the real rates in the event of an inflationary shock. A larger value of $\rho_r$ reduces the response to future inflation for a given choice of $\beta_\pi$ and therefore contributes to determinacy.

Third, the region for determinacy is smaller for the UIF rule than for the PCF rule. Indeterminacy is evident for some choices of $\{\beta_\pi, \rho_r\}$ starting at a horizon of five quarters, with regard to the UIF rule, and at six quarters for the PCF rule. In general, the region of determinacy is larger for the PCF rule than for the UIF rule.

4.4. Optimality

The policymaker chooses the triplet $\{H, \beta_\pi, \rho_r\}$ so as to produce a determinate rational expectations equilibrium and minimize the expected loss, as stated in equation (24). We use a grid-search over the values of the triplet producing a determinate RE equilibrium with a mask-
width of 0.05 for $\rho_r$ and 0.25 for $\beta_\pi$. We assume the policymaker to be a flexible inflation targeter in setting $\lambda = 0.50$ and $\nu = 0.25$ in (23). The optimal coefficients at different horizons and the associated standard deviations and loss are presented in Table 1.

### Table 1
Unconditional standard deviations in percent and losses.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Variables</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$\beta_\pi$</td>
<td>$\rho_r$</td>
</tr>
<tr>
<td><strong>UIF rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>0.95</td>
</tr>
<tr>
<td>6</td>
<td>6.00</td>
<td>0.75</td>
</tr>
<tr>
<td>7</td>
<td>6.25</td>
<td>0.90</td>
</tr>
<tr>
<td>8</td>
<td>6.25</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>12</td>
<td>1.00</td>
<td>0.05</td>
</tr>
</tbody>
</table>

| **PCF rule** | | | | | | | |
| 0 | 1.00 | 0.95 | 2.97 | 9.69 | 0.15 | 31.99 | 51.32 |
| 2 | 1.00 | 0.95 | 3.24 | 9.49 | 0.15 | 32.31 | 50.28 |
| 4 | 1.00 | 0.95 | 4.41 | 8.73 | 0.16 | 33.67 | 47.82 |
| 6 | 4.50 | 0.10 | 3.42 | 8.28 | 2.30 | 33.09 | 41.42 |
| 7 | 8.75 | 0.65 | 3.55 | 8.10 | 0.98 | 33.44 | 39.35 |
| 8 | 10.00 | 0.80 | 3.45 | 8.26 | 0.79 | 33.22 | 40.24 |
| 10 | 7.00 | 0.90 | 3.01 | 9.60 | 0.50 | 31.47 | 50.69 |
| 12 | 2.50 | 0.95 | 5.58 | 14.83 | 0.34 | 26.64 | 125.51 |

**Optimal commitment rule** $\lambda = 0.50$ $\nu = 0.25$

| | | | | | | |
| --- | --- | --- | ----- | ----- | --- |
| 4.90 | 5.70 | 3.26 | 36.60 | 30.9 |

---

a The first three columns show the optimal configuration for the policy rules at different forecast-feedback horizons. The next five columns show the standard deviations of the respective variables, and the last two columns show the expected losses.

Several results are worth noting. First, we find the optimal forecast-feedback horizon to be between one and two years. The optimal forecast-feedback horizon for the UIF rule is only one quarter shorter than for the PCF rule. Moreover, the optimal coefficients and the properties for the two rules are very similar at short horizons. Given the high degree of inertia in the model, the difference between the assumption of an unchanged-interest-rate and a policy-consistent inflation forecast is minor, if the forecast-feedback horizon is relatively short. For horizons above seven quarters, there are important differences, however. The optimal coefficients differ
significantly, with the policy-consistent rule being the most “active”. The PCF rule weakly dominates the UIF rule at all horizons considered, and the consequences of a long forecast-targeting horizon are more severe for the UIF rule, with strong volatility in inflation and output. In this respect, the PCF rule is the more robust to the choice of forecast-feedback horizon.

The optimal choice of coefficients involves considerable interest-rate inertia at most horizons, with $\rho_r$ being close to unity. The optimality of interest-rate inertia in forward-looking models is discussed in Woodford (2003b). Such inertia, or history-dependence, influences private-sector expectations about the future as a given monetary-policy stance is expected to prevail. The ability to affect expectations about the future will enhance the central bank’s ability to influence private-sector behavior today, because agents act in a forward-looking manner.

Inflation targeting is associated with extensive use of the exchange-rate channel. Real exchange-rate volatility is slightly smaller for the forecast-feedback rules as compared to the optimal commitment policy, so that the source of the volatility is not the forecast-feedback rules. Price stickiness on imported goods is known to lead to more exchange-rate variability under inflation targeting (see Adolfson, 2001); since prices are subject to stickiness, imported goods prices respond less to any shocks causing a movement in the exchange rate. Hence, exchange rate stability is not such an important requirement for inflation and output stability as it is if prices on the imported goods are flexible.

The optimized forecast-feedback rules generate a loss as much as 30 per cent worse than the optimal commitment policy. Relative to this policy, the forecast-feedback rules generate too much inflation stabilization and interest-rate smoothing, and too little output-gap stability. This result confirms and extends the result in Levin et al. (2003) for the relatively closed US economy. They argue that the performance of the rule can be significantly improved if the rule is extended with a response to the output gap. In this regard, neither rules is as “output encompassing” as claimed by Batini and Haldane (1999).

To further illustrate this, Table 2 shows the optimal horizon and loss relative to the optimal commitment policy for different configurations of $\lambda \in \{0.2, 0.6, 0.8\}$. It can be seen that both rules can bring the outcome close to the optimal commitment policy outcome, if the central bank is relatively strict on inflation, the loss being only 2.07 and 1.67 per cent worse for the UIF and PCF rules, respectively. However, the efficiency of both rules deteriorates very quickly as the relative weight on the output gap increases. However, the optimal forecast-feedback horizon is around a year and a half for both rules, irrespective of the value of $\lambda$.

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12 The weight on interest-rate smoothing in the loss function has little impact on this result. Reducing the weight on interest-rate smoothing to $\nu = 0.1$ does not produce any significant changes in this result.
Table 2
Central bank preferences and optimal rule configurations.a

<table>
<thead>
<tr>
<th>λ</th>
<th>βx</th>
<th>ρr</th>
<th>H</th>
<th>L(λ, .25)</th>
<th>Lr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UIF rule</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>6.25</td>
<td>0.90</td>
<td>7</td>
<td>23.23</td>
<td>2.07%</td>
</tr>
<tr>
<td>0.6</td>
<td>7.75</td>
<td>0.80</td>
<td>6</td>
<td>46.43</td>
<td>48.67%</td>
</tr>
<tr>
<td>0.8</td>
<td>7.75</td>
<td>0.80</td>
<td>6</td>
<td>57.77</td>
<td>100.38%</td>
</tr>
<tr>
<td>PCF rule</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>4.25</td>
<td>0.45</td>
<td>7</td>
<td>23.14</td>
<td>1.67%</td>
</tr>
<tr>
<td>0.6</td>
<td>8.75</td>
<td>0.65</td>
<td>7</td>
<td>44.65</td>
<td>42.97%</td>
</tr>
<tr>
<td>0.8</td>
<td>8.75</td>
<td>0.65</td>
<td>7</td>
<td>55.24</td>
<td>91.61%</td>
</tr>
</tbody>
</table>

a The table shows the optimal rule configurations for different weights on inflation versus output stabilization. L denotes the expected loss and $L_r \equiv \frac{(L - L_c)}{L_c} 100$ the loss relative to the loss under optimal commitment policy, in percent.

5. Conclusion

This paper has evaluated inflation forecast-feedback rules in an estimated, micro-founded model of the UK economy. We find that these rules bring inflation and output close to the optimal policy inflation-output variance frontier, and that they are close to the optimal rule as long as the central bank puts sufficiently large weight on stabilizing inflation. This confirms results obtained in models of relatively closed economies. The optimal forecast-feedback horizon is surprisingly stable at six or seven quarters, and independent of the weight the central bank attaches to inflation versus output stabilization. A potential problem with both forecast-feedback rules is that neither rules do not necessarily ensure determinacy of the rational expectations equilibrium. This problem is especially acute at long forecast-feedback horizons where the set of rule parameters that creates determinacy is quite small.

We find that the forecast-feedback rule using an unchanged-interest-rate forecast of inflation in general decreases the parameter determinacy space, and does not improve on the performance of the rule. The implied dynamics of the rules do, however, show an important difference when the central bank applies a long forecast-feedback horizon.
Appendix

A. Analytical derivation of policy with the constant-interest-rate forecasts as the indicator

This subsection derives the UIF policy in a general dynamic model. An important and large class of dynamic models can be set in the following state-space form:

\[ Z_{t+1} = AZ_t + Br_t + \varepsilon_{t+1}, \]  
(A1)

where \( A \) is the companion matrix and \( B \) is a vector of interest-rate impact multipliers; \( Z \) is a vector of state variables. Using repeated substitutions, we can write the expected value of the state vector at time \( t + h \), formed at time \( t \) as

\[ Z_{t+h|t} = A^h Z_t + \sum_{i=1}^{h} A^{h-i} B r_{t+i-1|t}. \]

Provided that the interest rate level in the previous period is kept throughout the forecast-targeting horizon, we may write the unchanged-interest-rate forecast as

\[ Z_{t+h|t}(\bar{r}_{t-1}) = A^h Z_t + \sum_{i=1}^{h} A^{h-i} B r_{t-1}. \]  
(A2)

Assuming that the state vector includes the relevant variables, we can set

\[ \bar{\pi}_t \equiv K_{\pi} Z_t, \]  
(A3)

\[ r_{t-1} \equiv K_r Z_t, \]  
(A4)

for appropriately defined \( K_{\pi} \) and \( K_r \). Then, we can insert (A3) and (A4) into (A2) to obtain the unchanged-interest-rate forecast of the four-quarter inflation rate as

\[ \bar{\pi}_{t+h|t}(\bar{r}_{t-1}) = K_{\pi} Z_{t+h|t}(\bar{r}_{t-1}) = K_{\pi} A^h Z_t + K_{\pi} \sum_{i=1}^{h} A^{h-i} B K_r Z_t. \]  
(A5)

The inflation forecast feedback rule is given from (1),

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) \beta_{\pi} \bar{\pi}_{t+h|t}(\bar{r}_{t-1}), \]

where \( \pi^* = 0 \). Using (A3), (A4) and (A5), this rule may be written as a function of the state vector as

\[ r_t = F Z_t, \]  
(A6)
where $F = \rho_r K_r + (1 - \rho_r) \beta_\pi K_\pi A^h + (1 - \rho_r) K_\pi \sum_{i=1}^h A^{h-i} BK_r$.

If the state vector consists of both backward-looking, $z_{1,t}$, and forward-looking, $z_{2,t}$, variables, i.e., $Z_t = \begin{bmatrix} z_{1,t} & z_{2,t} \end{bmatrix}'$, then the state space form in (A1) may be written as,

$$\begin{bmatrix} z_{1,t+1} \\ z_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} r_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{A7}$$

The interest rate rule in (A6) may be written as $r = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix}$. After inserting the interest rate rule in (A7), we obtain

$$\begin{bmatrix} z_{1,t+1} \\ z_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

$$= \left( \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + \begin{bmatrix} B_1 F_1 & B_1 F_2 \\ B_2 F_1 & B_2 F_2 \end{bmatrix} \right) \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}, \tag{A8}$$

where the $C$-matrix is accordingly defined.

Blanchard and Kahn (1980) show the rational expectations equilibrium to be unique if the number of eigenvalues of $C$ outside the unit circle is equal to the number of forward-looking variables. The forward-looking variables may then be written as a linear function of the predetermined variables, $z_{2,t} = V z_{1,t}$. \tag{A9}

The the matrix $V$ is determined of the underlying model coefficient, and can be found by applying the techniques discussed Klein (2000) and Söderlind (1999).

Note that, in equilibrium, the interest rate follows

$$i_t = (F_1 + F_2 V) z_{1,t}.$$
References


- 2002, The unique minimum state variable RE solution is e-stable in all well formulated linear models, manuscript, Carnegie Mellon University.


Rudebusch, Glenn, 2002a, Term structure evidence on interest rate smoothing and monetary policy inertia, Journal of Monetary Economics 49, 1161–1187.


Sveriges Riksbank 1999 Inflation Report no. 3 Sveriges Riksbank.

